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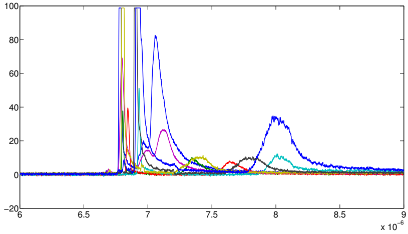
Quantifying Uncertainty in Plasma Temperature via Neutron Detection



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Joint work with Michal Odyniec, Eric Machorro, John Bardsley, Heikki Haario, and others…

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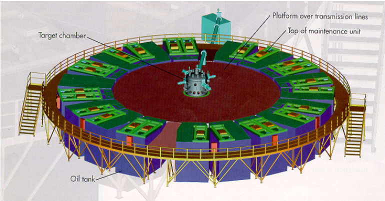
National Security Technologies, LLC (NSTec), is the Management & Operations Contractor to the U.S. Department of Energy for the Nevada National Security Site.

We develop and field experimental diagnostics such as

- X-ray Radiography,
- High Speed Cameras,
- Photonic Doppler Velocimetry,
- VISAR, and
- Radiation Detectors



http://www.nv.energy.gov/library/photos/atlas.aspx

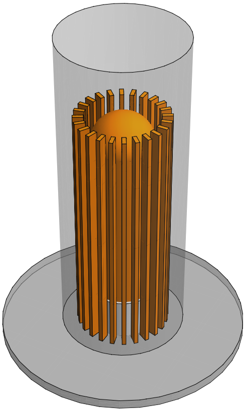
We also develop methods for analyzing and interpreting the data from these diagnostics.

Neutron Tomography

Primary Problem: Determine the temperature of a plasma in controlled fusion reactions.

What do we mean by controlled fusion?

Small amounts of Deuterium or Deuterium and Tritium get compressed to the point of fusing, emitting photons and neutrons.



Dense Plasma Focus Schematic, compliments of Tim Meehan.

Radiation Detectors

Neutron Detectors:

1. Neutron interacts with scintillator, which releases a charged particle,
2. which bounces around and releases photons,
3. into a photomultiplier tube.
4. Released photons hit the photocathode,
5. generating a flow of electrons.

Notes:

- Neutrons are counted regardless of energy.
- Neutrons have mass, so higher energy neutrons are faster than low energy neutrons.
- Scatter effects are difficult to distinguish from the desired interactions.

$**$ Detector signal as a function of detection time t' and distance X is the integral of neutron flux over creation time t and energy E, which is a line integral in creation time and inverse velocity coordinates.$**$

Given creation time t and energy E, the neutron profile can be modeled as a Gaussian

$$f(q,t) = \frac{N}{2\pi\sigma_q\sigma_t} \exp\left(\frac{-(q-1)^2}{2\sigma_q^2} - \frac{(t-t_0)^2}{2\sigma_t^2} \right), \qquad (1)$$

where

- N - Total number of neutrons created in reaction,
- q - Energy represented in inverse velocity coordinates,
- σ_q^2 - variance of neutron inverse velocity profile, which is proportional to plasma temperature, which is what we really care about, and
- σ_t^2 - variance of neutron creation time profile.

Radon Transform

The detector signal is thus the Radon transform of the distribution $f(q,t)$,

$$
\begin{aligned}
(Rf)(X,t') &= \frac{N}{2\pi\sigma_q\sigma_t}\int_{\mathbb{R}}\int_{\mathbb{R}} e^{\frac{-(q-1)^2}{2\sigma_q^2}}\, e^{\frac{-(t-t_0)^2}{2\sigma_t^2}}\,\delta(t-(t'-Xq))\,dt\,dq \\
&= \frac{N}{X^2\sqrt{2\pi\sigma^2}}e^{\frac{-(t'-(X+t_0))^2}{2\sigma^2}}, \qquad (2)
\end{aligned}
$$

where $\sigma^2 = \sigma_q^2 X^2 + \sigma_t^2$, and we use a specialized coordinate system that makes the algebra a little easier.

Optimization Problem

The optimization problem is to compute the model parameters σ_q^2, σ_t^2, and t_0, by fitting the model

$$S_j(X_j, t') = \kappa_j \frac{N\alpha_j A_j}{4\pi X_j^2 \sqrt{2\pi\sigma^2}} e^{\frac{-(t' - (X_j + t_0))^2}{2\sigma^2}} \tag{3}$$

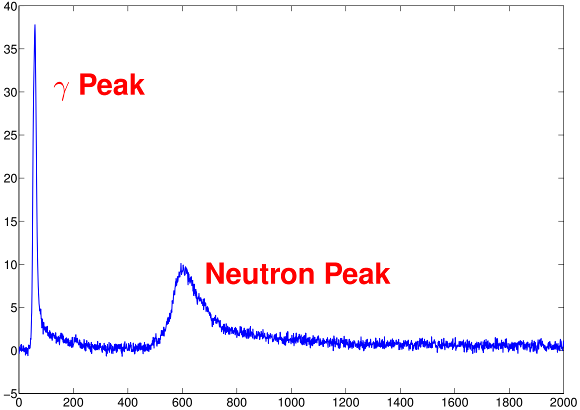
to the data measured at detectors $1, \ldots, J$, where

- α_j - sensitivity of the detector,
- A_j - detection surface area,
- κ_j - multiplicative factor for transforming particles to voltage (also a model parameter, just not important),
- $\sigma^2 = \sigma_q^2 X^2 + \sigma_t^2$.

Notes and Primary Questions:

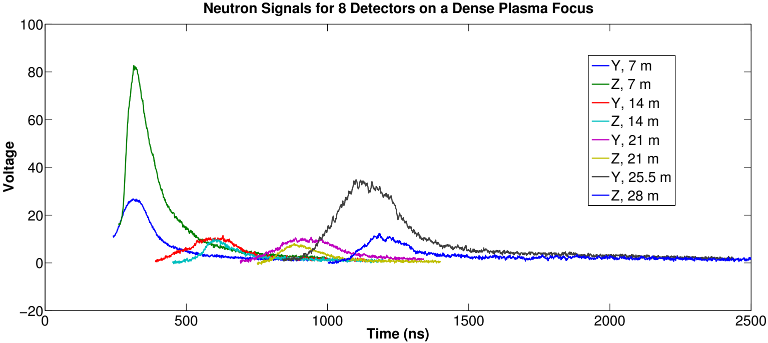
- The data are sparse in X but (fairly) dense in t'. Usually have 3-8 detectors.
- At least 2 detectors at different distances are necessary to resolve σ_q^2 and σ_t^2.
- The data from the detectors is often inconsistent with itself.
- What is the appropriate noise model? It should have Poisson and Gaussian components, but the noise seems to vary significantly.
- Given the ambiguity in the noise model, what are appropriate signal reconstruction techniques?

Total Signal



Signal from radiation detector measuring the NSTec Dense Plasma Focus.

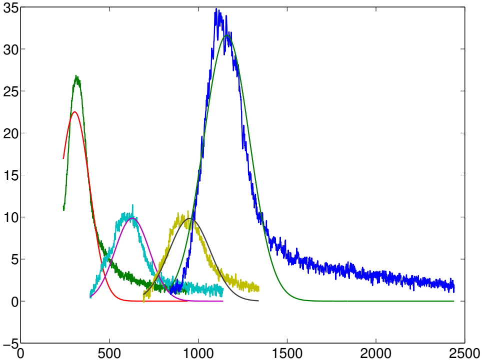
Neutron Signals



Neutron signals from 8 detectors placed at 7, 7, 14, 14, 21, 21, 25.5, and 28 m from the source.

Nonlinear Least Squares Results

4 of the 8 detectors shown in the previous slide, and nonlinear least squares "optimized" estimates for the model parameters.



Optimized Parameters: $\kappa_1 = 0.0006$, $\kappa_2 = .00121$, $\kappa_3 = 0.0032$, $\kappa_4 = 0.0171$, $t_0 = -19.17$, $\sigma_q^2 = 0.0075$, $\sigma_t^2 = 6398.6$.

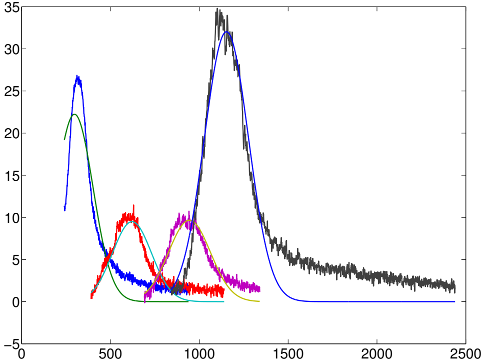
Nonlinear LSq Issues

- Highly sensitive to initial guesses for parameters: “solution regimes,”
- Also sensitive to the number of detectors used and how self-consistent the detector data is,
- No natural method for estimating "error bars" on the parameter values,
- Assumes a Gaussian noise model, which is not really our case, unless using a weighted least squares, which can be difficult to formulate as an MLE for any realistic error model.

MCMC Estimation

Alternate approach is to use a Markov Chain Monte Carlo (MCMC) approach...

The same 4 detectors as shown above with MCMC optimized model parameters.



MCMC optimization computed using the MCMCStat Toolbox for Matlab, developed by Haario and colleagues.

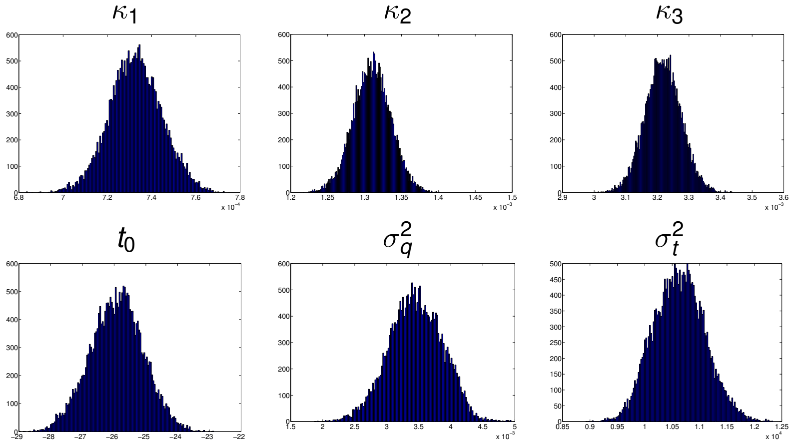
MCMC Results

There are also some issues with using the MCMC approach:

Parameter	Median	Mean	Rel. Std.
κ_1	0.00135	0.00135	0.0228
κ_2	0.00325	0.00325	0.018
κ_3	0.0165	0.0165	0.0065
t_0	-28.29	-28.26	0.03
σ_q^2	0.00192	0.00191	0.427
σ_t^2	12541	12541	0.086

Note the parameter about which we care the most, σ_q^2, is also the parameter whose estimate we trust the least. These values are also highly sensitive to the self-consistency of the detector data.

MCMC Posterior Distributions



Histograms of the “ends” of the MCMC chains for 6 of the 7 model parameters. (The 4th κ just didn’t fit on the slide, but it looks like the others.)

Issues with MCMC Approach

- Also sensitive to initial guesses,
- Requires some notion of initial distributions on parameters,
- Not as robust to inconsistencies in the data,
- The posterior distributions may just be a function of the error function and the assumed prior distributions, rather than really meaningful distributions for error estimation.
- A Pro: a variety of objective functions can be used to compute the chains in order to treat different noise models.

Where we stand...

- This is just one of several parameter estimation problems that we have in order to extract meaningful information from real data.
- We also have non-parametric signal reconstruction problems that we solve and are working to incorporate systematic UQ.
- We're starting to work with John and Heikki on some further methodology, but
- we are happy to take suggestions for approaches.

Thank you!

- We thank Tim Meehan for the DPF schematic and Chris Hagen and the entire NSTec DPF team for the DPF data and for helpful comments and suggestions on the physics of controlled fusion.
- Feel free to contact me if you have any questions or comments, or for references, offprints, and preprints.
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