Quantified uncertainties: the good, the bad and the plain ugly

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OCMO

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Outline

1. Threads
2. Chapter 1
3. Spectrum of uncertainty assessment
4. Least squares and UQ
5. Canonical geophysical example
6. An illuminating very low dimensional problem
7. Discussion
Motivation, 10 years ago

- Didn’t think the people I was working with really knew much about probability (but that didn’t stop them using the language)
- Markets exist independently of each other but there is both commonality and exclusivity:
  - Lloyds (catastrophic physical risks)
  - London Stock Exchange
  - Derivative securities
- Lest we forget–excess of loss spiral in Lloyds, 2008 crash
Otago Regional Council’s water quality plan changes

Proposed limit of 10kg of nitrogen per hectare of land per year, on average, going into groundwater.
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Disgruntled farmer

‘. . . let’s base it on science rather than assumptions or modelling. . . ’. North Otago farmer Robert Borst.
Otago Regional Council’s water quality plan changes

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Disgruntled farmer

‘...let’s base it on science rather than assumptions or modelling...’ North Otago farmer Robert Borst.

ORC response

‘...modelling was science in action.’ ORC policy director Fraser McRae.
The Challenges

- How do you sell inverse problems and UQ?
- Regularisation does very nice, thank you!
- If you play the UQ card you open up a can of worms
- Stakeholder buy-in
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First selling point:

- Measurement uncertainty
- Model uncertainty

Therefore, need to quantify **envelope of uncertainty**
Aims

- to be disagreeable!
- to remind us of our responsibilities to engage a broader debate (Sir Paul Callaghan, FRS)
- to invite robust comment
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<th>Hayman’s question</th>
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<td>‘What is that number ( p ) between 0 and 1?’ (Prof W.K. Hayman, FRS, Imperial College 1960’s)</td>
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Chapter 1

### Hayman’s question

‘What is that number $p$ between 0 and 1?’ (Prof W.K. Hayman, FRS, Imperial College 1960’s)

### Law(s) of averages

- Basic intuition wrapped up in ideas of **repeatability** and **regularity**: repeat an experiment indefinitely with $n$ different outcomes (or events). Then the proportion of occurrences of a particular event will ‘approach’ the probability of that event.

- This is what makes probability a useful tool at all levels.

- In particular: ‘return period’ (engineers, underwriters)

- No good trying to hide under Kolmogorov’s shirt tails
‘Mathematics (by which I shall mean pure mathematics) has no grip on the real world; if probability is to deal with the real world it must contain elements outside mathematics; the meaning of probability must relate to the real world, from which we can then proceed deductively (i.e. mathematically). We will suppose (as we may by lumping several primitive propositions together) that there is just one primitive proposition, the ‘probability’ axiom, and we call it $A$ for short. Although it has got to be true, $A$ is by the nature of the case incapable of deductive proof, for the sufficient reason that it is about the real world ... ’
From a Mathematician’s Miscellany:

‘Mathematics (by which I shall mean pure mathematics) has no grip on the real world; if probability is to deal with the real world it must contain elements outside mathematics; the meaning of probability must relate to the real world, from which we can then proceed deductively (i.e. mathematically). We will suppose (as we may by lumping several primitive propositions together) that there is just one primitive proposition, the ‘probability’ axiom, and we call it A for short. Although it has got to be true, A is by the nature of the case incapable of deductive proof, for the sufficient reason that it is about the real world . . . ’

Back to: ‘What is that number \( p \) between 0 and 1?’

- Mathematical theory gives no prescription for assigning probabilities
- Sufficient that some self evident axioms are satisfied
1 Threads

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Earthquake forecasting

**USGS earthquake forecast in SF Bay area**

David Freedman’s analysis in ‘What is the probability of an earthquake?’
USGS earthquake forecast in SF Bay area

David Freedman’s analysis in ‘What is the probability of an earthquake?’

Stage 1
2,000 models to predict rate of tectonic deformation
Earthquake forecasting

USGS earthquake forecast in SF Bay area
David Freedman’s analysis in ‘What is the probability of an earthquake?’

Stage 1
2,000 models to predict rate of tectonic deformation

Monte Carlo

Stage 2
3 stochastic models for fault segment ruptures

Deduction
Prob of earthquake 0.7 ± 0.1 (OCMO)
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Earthquake forecasting

USGS earthquake forecast in SF Bay area

David Freedman’s analysis in ‘What is the probability of an earthquake?’

**Stage 1**
2,000 models to predict rate of tectonic deformation

Monte Carlo

**Stage 2**
3 stochastic models for fault segment ruptures

**Deduction**
Prob of earthquake 0.7 ± 0.1
Freedman’s views

- ‘by a process we do not understand, those uncertainties [estimated in stage 1] were propagated through stage 2 to estimate the uncertainty of the estimated probability of a large earthquake. If this view is correct, 0.1 is a gross underestimate of the uncertainty’

- 10 sources of error overlooked
Freedman’s views

- ‘by a process we do not understand, those uncertainties [estimated in stage 1] were propagated through stage 2 to estimate the uncertainty of the estimated probability of a large earthquake. If this view is correct, 0.1 is a gross underestimate of the uncertainty’
- 10 sources of error overlooked

Morals

- Don’t get seduced by Monte Carlo sampling!
- Action is a poor apology for thought (Milne Anderson, UCL)
- Assigning a probability gives a cloak of respectability
What is the risk of liquefaction?

- Simplified method
- Process:

1. Generate excess pore water pressure through cyclic loading
2. Loss of stiffness (due to reduction in effective stress)
3. Liquefaction
Liquefaction assessment: simplified method

EQ load

Liquefaction resistance

Cyclic stress ratio

$$CSR = \frac{\tau_{\text{cyc}}}{\sigma'_{v}} = 0.65 \frac{a_{\text{max}}}{g} \frac{\sigma_{v}}{\sigma'_{v}} r_{d}$$

amplitude, (frequency?)

Cyclic resistance ratio

$$CRR = CRR_{M=7.5} \times MSF \times K_{\sigma}$$

If

$$F_{L} = \frac{CRR}{CSR} < 1$$

then liquefaction
Sources of uncertainty

Obvious sources of uncertainty

Even if you accept the procedure there are numerous uncertainties with the scalings:

- 0.65
- Stress reduction factor, $r_d$ (many empirical formulae)
- Magnitude scaling factor, MSF
- Effects of overburden stress $K_\sigma$

If based on lab tests:

$$CRR_{\text{field}} = C_1 C_2 C_3 C_4 C_5 CRR_{\text{triaxial}}$$

As well as with other corrections for soil type, sloping ground &c., and the all important interpretation of test data

Then there are harder questions about the underlying methodology:

- Lab inspired approach to analysis, over regularising (?), the ‘uniform assumption’, equivalent number of cycles
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Example 1: Linear least squares fit with good UQ
Example 2: Linear least squares fit with poor UQ
What went wrong?

Example 1
Model calibrated, i.e. 'fitted to data', confidence intervals reasonable
Parameter uncertainty reasonable, i.e. capture true values
Predictive uncertainty reasonable, i.e. capture reality

Example 2
Model calibrated
But
- poor parameter uncertainty, i.e. don't capture true values
- poor predictive uncertainty

Distinction
Good and bad UQ

(OCMO)
What went wrong?

Example 1
- Model calibrated, i.e. ‘fitted to data’, confidence intervals reasonable ✓
- Parameter uncertainty reasonable, i.e. capture true values ✓
- Predictive uncertainty reasonable, i.e. capture reality ✓

Example 2
- Model calibrated ✓
- But
  - poor parameter uncertainty, i.e. don’t capture true values ✗
  - poor predictive uncertainty ✗
What went wrong?

Example 1
- Model calibrated, i.e. ‘fitted to data’, confidence intervals reasonable ✅
- Parameter uncertainty reasonable, i.e. capture true values ✅
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Example 2
- Model calibrated ✅
- But
  - poor parameter uncertainty, i.e. don’t capture true values ❌
  - poor predictive uncertainty ❌

Distinction
- Good and bad UQ
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Scenario

Lake Ellesmere
Rakaia River
Waimakariri R.
Banks Peninsula
Southern Alps

(a)

(b) XY Cross Section

Concealed fault

Greendale Fault

Canterbury Plains

Fig. 5

CHRISTCHURCH

43°30'S

20 km

2472000 2476000 2478000 2484000 2488000

Easting (NZMG)

X Y

40 0

-40

-80

-120

m asl

XY Cross Section

(5743000N)
Test case

Confined Conditions
T/S large

Unconfined Conditions
T/S small

X (m)
Pumped Well

Y (m)

P1
P2
P3
P4
O1
O2
O3
O4

Confined Conditions

Unconfined Conditions

Quantified uncertainties
Problem

Model

\[
S(x, y) \frac{\partial s(x, y, t)}{\partial t} = \nabla \cdot (T(x, y) \nabla s(x, y, t)) + Q \delta(x) \delta(y) \quad \text{in } \Omega \tag{1}
\]

\[
s(x, y, t) = 0 \quad \text{on } \partial \Omega \tag{2}
\]

\[
s(x, y, 0) = 0 \tag{3}
\]

where \( s = s(x, y, t) \) is drawdown, \( T = T(x, y) \) and \( S = S(x, y) \) are spatially distributed transmissivities and storativities, and \( Q \) is the constant pumping rate over the duration of pumping.

Problem

- Carry out a pump test and observe drawdown in bores O1-O4.
- Estimate distributed log-transmissivity \( \log T = \log T(x, y) \) and log-storativity \( \log S = \log S(x, y) \)
What can we reasonably expect to reconstruct from the data from bores O1-O4?

Presumably, there will be a zone where the data tells us something, outside the data will give little/no information.

What is a sensible prior model?

- what structural information does it make sense to use?
- what scales can we expect capture?
Draws from prior

(a) x, y

(b) x, y

(c) x, y

(d) x, y

Quantified uncertainties
Data

(a) Observation Well O1

(b) Observation Well O2

(c) Observation Well O3

(d) Observation Well O4
Reconstruction

(a) x
(b) x
(c) y
(d) y
Calibration

(a) Observation Well O1

(b) Observation Well O2

(c) Observation Well O3

(d) Observation Well O4

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Prediction at measurement points

(a) Observation Well O1

(b) Observation Well O2

(c) Observation Well O3

(d) Observation Well O4
Prediction at test points P1-P4

(a) Test Well P1

(b) Test Well P2

(c) Test Well P3

(d) Test Well P4
Longitudinal cut through parameter reconstruction

(a)  
(b)  

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Quantified uncertainties
Pump test data

Quantify stream depletion
Unbounded uncertainty

Plots of log-likelihood as functions of $T_1$ and $\lambda$ (with all other parameters true values)

- (a) uses $d_2$, (b) $d_1$ and $d_2$
- Seems $\lambda$ somewhat ill-determined by data
Data for shallow pumping test

drawdown in water table

data $d_3$

true

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Posterior parameter uncertainty using data $d_2$ and $d_3$
Uncertainty in stream depletion using data $d_2$ and $d_3$

- At least stream depletion bounded
- Though would grossly overestimate
Posterior parameter uncertainty using all data $d_1 - d_3$
Uncertainty in stream depletion using all data $d_1 - d_3$

- More acceptable
- Mildly overestimates
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Discussion: Key references

- Cui and Dudley Ward (2012), Uncertainty quantification for stream depletion tests, J.Hydr.Eng
- Cui, Dudley Ward & Kaipio (2012), Characterisation of aquifer parameters from pumping test data for a heterogenous aquifer, Under review, J.Hydr.Eng
- Dudley Ward (2012), The Book, MS.
Why do quantified uncertainty?

- Insufficient consideration of uncertainty leads to poor predictive reliability
- Good UQ essential to decision making, since quantified risks can be controlled
- But: bad UQ is unhelpful at best, positively misleading at worst
Discussion: Summary

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Take home advice:

- Beware the pitfalls of Monte Carlo sampling
Discussion: Summary

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To reflect on:

- Effective public communication of risk
- See http://understandinguncertainty.org