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In-Situ Measurement of Power Flow and Mechanical Properties of Vibrating Timber Structures.

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- Bending waves in a beam
- Measure wave amplitudes
- and effective mechanical properties

Bending waves in a beam

Thin-beam model

$$B\eta_{xxxx} + m\eta_{tt} = p$$

$\eta(x,t)$ is the transverse displacement

$p(x,t)$ is the applied pressure

B is the *effective* bending stiffness of the beam

m is the *effective* mass per unit length

Modes: $\mathrm{e}^{i(kx+\omega t)}$

$$k = (\omega^2 m/B)^{1/4}$$

away from forcing, joints

$$\eta(x,t) = \mathrm{e}^{i\omega t}\left(a_1\mathrm{e}^{ik_\omega x} + a_2\mathrm{e}^{k_\omega x} + a_3\mathrm{e}^{-ik_\omega x} + a_4\mathrm{e}^{-k_\omega x}\right)$$

$\eta(x,\omega) = (\cdots)$

a_1, a_3 coefficients of waves travelling towards $-\infty$ and $+\infty$

other two modes are evanescent

Power Flow: $P = |a_1|^2 B\omega k^3 = |a_1|^2 B^{1/4} m^{3/4} \omega^{5/2}$

In Situ Measurement

Measure complex amplitudes $\eta(x_l, \omega)$ at positions, $x_1, x_2, \cdots, x_N$ (actually $-\omega^2\eta(x_l, \omega)$)

Estimating modal amplitudes

When m/B is known, find $a = (a_1, a_2, a_3, a_4)^T$ by solving

$$Ea = y$$

where

$$\begin{aligned} E &= \begin{pmatrix} \mathrm{e}^{ik_\omega x_1} & \mathrm{e}^{k_\omega x_1} & \mathrm{e}^{-ik_\omega x_1} & \mathrm{e}^{-k_\omega x_1} \\ \mathrm{e}^{ik_\omega x_2} & \mathrm{e}^{k_\omega x_2} & \mathrm{e}^{-ik_\omega x_2} & \mathrm{e}^{-k_\omega x_2} \\ \vdots & \vdots & \vdots & \vdots \\ \mathrm{e}^{ik_\omega x_N} & \mathrm{e}^{k_\omega x_N} & \mathrm{e}^{-ik_\omega x_N} & \mathrm{e}^{-k_\omega x_N} \end{pmatrix} \\ y &= (\eta(x_1, \omega), \eta(x_2, \omega), \cdots, \eta(x_N, \omega),)^T \end{aligned}$$

E generally invertible for $N = 4$, though ill-conditioned over a range of frequencies

Improve accuracy using more than 4 measurement locations

$$\hat{a} = \left(E^H E + \alpha I\right)^{-1} E^H y,$$

Maximum likelihood estimation when measurement error is additive, i.i.d. zero-mean normal, assume that the modelling error can be treated within the same framework

Estimating the mechanical properties

When measurements are made at 5 or more locations, extend the method given above to include estimation of the ratio m/B

$$(a_1, a_2, a_3, a_4, m/B) = \arg\min \|Ea - y\|_2^2$$

$$m/B = \arg\min \|E\left(E^H E + \alpha I\right)^{-1} E^H y - y\|_2^2$$

Second measure required to obtain values B and m separately. Use point forcing, ratio of force to amplitude of the outward travelling wave depends on the factor $m^3 B$, measuring this ratio allows both parameters to be determined.

Optimal measurement location

Choose measurement locations to optimize Fisher information in measurements about m, B

Assuming Gaussian noise statistics

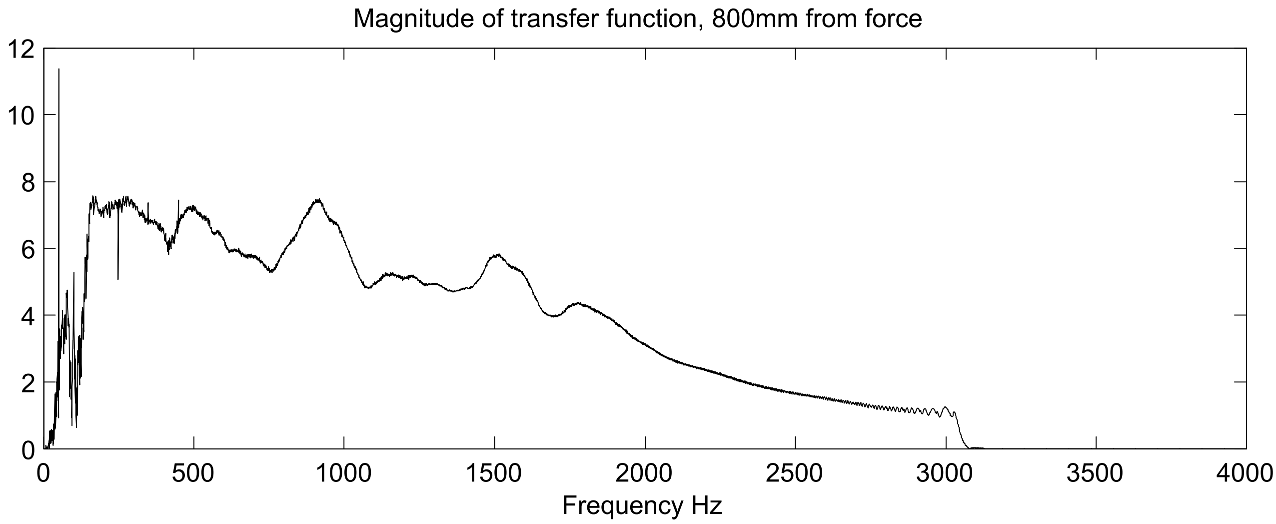
Misfit $\|Ea - y\|_2^2$ proportional to the log-likelihood for parameters given measurements y

$$(x_1, x_2, \cdots, x_N) = \arg\max \|\frac{\partial}{\partial(m/B)} Ea\|_2^2$$

Maximize this number over feasible measurement positions.

Experimental Results

- 2.8 m length of 100 mm $\times$ 50 mm dry pinus radiata mounted between sand traps.
- Beam centrally driven by a point source with power between 100 Hz and 3 kHz.
- Resulting transverse acceleration measured at distances (close to) 400 mm, 600mm, 800 mm, 900 mm, and 1000 mm from forcing



Fited modal amplitudes and m/B

- Simple sand trap achieved about 99 % energy absorption
- B/m decreases from 2.5×10^3 Nm^2 at 100 Hz to 1.45×10^3 Nm^2 at 3 kHz, roughly linearly with frequency.
- Compare with 2.65×10^3 Nm^2 measured statically.

Conclusions

- Both modal amplitudes and mechanical properties can be estimated from measurements of transverse motion.
- Effective mechanical properties of pinus radiata vary with frequency
- Hence, accurate measurement of power flow cannot rely on the statically measured value of m and B

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