Characterising radiation-belt energetic electron precipitation spectra: a comparison of quasi-linear diffusion theory with in-situ measurements

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Key Points:

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9		Decreasing the cold plasma density increases calculated electron precipitation at
10		the higher energies.
11	•	Calculated precipitation for a low Earth orbiting detector is weakly dependent on
12		the source spectrum.
13	•	RBSP derived diffusion coefficients provide more diffusion for precipitating par-
14		ticles than those derived from a larger wave database.

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15 Abstract

High energy electron precipitation from the Earth's radiation belts is important 16 for loss from the radiation belts and atmospheric chemistry. We follow up investigations 17 presented in Reidy et al. (2021) where precipitating flux is calculated inside the field of 18 view of the POES T0 detector using quasi-linear theory and pitch angle diffusion coef-19 ficients $(D_{\alpha\alpha})$ from the British Antarctic Survey (BAS). These results showed good agree-20 ments at >30 keV for L^{*} >5 on the dawnside but the flux were too low at higher en-21 ergies. We have investigated the effect of changing parameters in the calculation of the 22 23 precipitating flux to improve the results for the higher energies using comparisons of insitu flux and cold plasma measurements from GOES-15 and RBSP. We find that the strength 24 of the diffusion coefficients rather than the shape of the source spectrum has the biggest 25 effect on the calculated precipitation. In particular we find decreasing the cold plasma 26 density used in the calculation of $D_{\alpha\alpha}$ increases the diffusion and hence the precipita-27 tion at the loss cone for the higher energies, improving our results. The method of cal-28 culating $D_{\alpha\alpha}$ is also examined, comparing co-located rather than averaged RBSP mea-29 surements. We find that the method itself has minimal effect but using RBSP derived 30 $D_{\alpha\alpha}$ improved our results over using $D_{\alpha\alpha}$ calculated using the entire BAS wave data base; 31 this is potentially due to better measurements of the cold plasma density from RBSP 32 than the other spacecraft included in the BAS wave data base (e.g. THEMIS). 33

³⁴ Plain Language Summary

High energy particles trapped in the Earth's radiation belt can enter the atmosphere, 35 known as particle precipitation, and collide with atmospheric particles, which can change 36 the atmospheric chemistry. This input into our atmosphere is key to understanding the 37 effects of space weather on our climate system variability but is difficult to quantify. Reidy 38 et al. (2021) calculated the precipitation that would be measured by a low-Earth orbit-39 ing satellite using wave-particle theory and diffusion coefficients from a radiation belt 40 model. Diffusion coefficients describe the amount of diffusion of the trapped radiation 41 belt particle population driven by different sources (e.g. chorus waves). Reidy et al. (2021) 42 found good agreement between the calculated and measured precipitation for lower en-43 ergy particles but found there was something missing for the higher energies. This paper investigates the impact of changing certain parameters within the calculations, find-45 ing the cold plasma density to be key in improving the results at higher energies. 46

47 **1** Introduction

High energy electron precipitation plays a significant role within magnetospheric 48 dynamics, both as a mechanism of loss from the Earth's radiation belts and by the im-49 pact on the atmospheric chemistry. Several attempts have been made to quantify this 50 input, using particle measurements from low-orbiting spacecraft, such as POES (e.g. Rodger, 51 Clilverd, et al. (2010); Nesse Tyssøy et al. (2016)) and from ground-based instrumen-52 tation (e.g. Rodger, Clilverd, et al. (2010); Rodger et al. (2013)). There have also been 53 attempts to quantify precipitation from radiation belt models (e.g. Jordanova et al. (2016); 54 Ferradas et al. (2019)). 55

Recently, Reidy et al. (2021) used wave-particle theory to calculate the precipitating flux that would be measured by the POES particle detector orientated towards local zenith (termed T0), these calculations required bounce averaged pitch angle diffusion coefficients and a source spectrum for the differential flux. Reidy et al. (2021) compared the calculated T0 flux to in-situ measurements from POES; these calculations showed good agreement on the dawnside for $L^* > 5$ for the >30 keV electron channel, as expected from using an averaged wave-model to generate the diffusion coefficients for chorus waves. However, these calculations significantly underestimated the >100 keV flux, by more than a factor of 10 in some MLT/L* sectors.

For the differential source spectrum, Reidy et al. (2021) fitted a kappa distribution, 65 assuming a spectral index of $\kappa = 5$, to the integral flux measurements from the POES 66 telescope aligned perpendicular to T0 (termed T90), making sure the telescopes field of 67 view was outside the loss cone and hence measuring trapped (or quasi-trapped) parti-68 cles (see Appendix A of Rodger, Carson, et al. (2010)). A kappa distribution was first 69 shown to be effective at modelling the particle distributions in the radiation belts by Summers 70 71 and Thorne (1991) and has subsequently been used in several studies to represent the differential flux spectrum (e.g. Li et al. (2013) and Glauert et al. (2018)). Whittaker et 72 al. (2013) found using DEMETER data that a spectral index of $\kappa > 2$ worked well for 73 fitting the distributions, with lower values of κ providing a harder spectrum. Whittaker 74 et al. (2013) also applied power-law and exponential fits to the DEMETER electron spec-75 tra, finding a power-law spectral gradient to consistently provide the best fit. Using Van 76 Allen Probes data during 2017, Zhao et al. (2019) found an exponential spectrum fit best 77 outside the plasmasphere, with a power law mostly occurring during injections at high 78 L^{*}, whilst flux inside the plasmasphere was dominated by bump-on-tail distribution due 79 to interactions with Hiss waves. The effect of these different types of spectral fit (i.e. power-80 law and exponential), as well as the impact of lowering the spectral index κ in the kappa-81 fit, on the calculated precipitation from Reidy et al. (2021) will be investigated in this 82 paper. 83

Most radiation belt models, such as the British Antarctic Survey Radiation Belt 84 model (BAS-RBM, Glauert et al. (2014)), solve a diffusion equation to quantify the evo-85 lution of flux within the radiation belts; wave-particle interactions are incorporated in 86 these equations by diffusion coefficients. Diffusion coefficients can be calculated using 87 statistical wave models, giving average diffusion coefficients based on averaged wave data 88 for different geomagnetic activity levels (e.g. Glauert and Horne (2005)) or from in-situ 89 data giving event specific diffusion coefficients (e.g. Ripoll et al. (2019)). One of the key 90 variables in diffusion coefficient calculations is the cold plasma density, which can alter 91 the electron energy and pitch angles at which resonant interactions occur. Allison et al. 92 (2021) found, using Radiation Belt Storm Probe (RBSP) in-situ waves and flux measure-93 ments, that decreases in the electron plasma density results in enhancements of the dif-94 fusion coefficients (both in energy and pitch angle) across all energy ranges. Allison et 95 al. (2021) show that during extreme depletion's of the plasma density, energy diffusion 96 due to chorus can be sufficiently high to accelerate electrons to >7 MeV energies. Allison 97 et al. (2021) also note a decrease in density would increase pitch angle diffusion near the 98 loss cone, thereby also increasing the loss from the radiation belts. 99

The method used to calculate diffusion coefficients has recently been examined; Watt 100 et al. (2019) found very different values of diffusion coefficients, calculated with the same 101 data sets, depending whether they were calculated from averaged values or if they were 102 calculated using co-located measurements of the wave spectra and f_{pe}/f_{ce} and then av-103 eraged. Ross et al. (2020) re-calculated EMIC diffusion coefficients using co-located wave 104 measurements rather than the averaged values and found better agreement when using 105 them in a radiation belt model (BAS-RBM) compared with RBSP data. Similarly, Wong 106 107 et al. (2022) found improvements for magnetosonic waves. This new method of calculating the diffusion coefficients with co-located data captures more variability of the sys-108 tem, allowing better representation of the extreme cases. Both Watt et al. (2019) and 109 Ross et al. (2020) suggest other diffusion coefficients, such as that for chorus, should be 110 re-calculated using similar techniques. 111

In this study we separately investigate the impact of the source spectrum, as well as two forms of variability within the diffusion coefficients, on the calculated precipitation using the same methods as in Reidy et al. (2021). In Section 2 we outline the instrumentation and methods used to evaluate the precipitation. In Section 3.1, we com-

pare the differential flux spectrum derived from POES T90 measurements (used as the 116 source spectrum for calculating precipitation in Reidy et al. (2021)), with in-situ differ-117 ential flux measurements made by GOES-15, when GOES-15 and the POES spacecraft 118 were in the same L^* and Magnetic Local Time (MLT) sector during March 2013. We then 119 investigate the impact of changing the shape of the source spectrum on the calculated 120 precipitation. In Section 3.2, we investigate the impact of the cold plasma density on the 121 amount of pitch angle diffusion at the loss cone, first by comparing the modelled f_{pe}/f_{ce} 122 with in-situ measurements from RBSP-A during November 2012 (a time when the RBSP 123 orbit was at high L-shell on the dawnside, between 06-08 MLT) and then by re-calculating 124 the chorus diffusion coefficients with f_{pe}/f_{ce} multiplied and divided by two. Lastly, in 125 Section 3.3 we re-calculate the chorus diffusion coefficients using RBSP data, firstly with 126 averaged measurements and then using co-located measurements in a similar way to Ross 127 et al. (2021). We present discussions and conclusions of these investigations in Sections 4 128 and 5 respectively. 129

¹³⁰ 2 Instrumentation and method

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2.1 Polar Orbiting Environmental Satellite (POES)

The POES constellation are low Earth orbiting satellites (800–850 km altitude), 132 in Sun-synchronous orbits. We are using data from the Medium Energy Proton and Elec-133 tron Detectors (MEPED) instrument, part of the Space Environment Monitor (SEM-134 2) package. MEPED has two electron solid state detectors, one centered 9° off local zenith 135 (T0) and the other perpendicular to this (T90). These instruments provide integral flux 136 measurements of the electrons between 30 and 2500 keV in three channels (>30, >100, 137 and >300 keV) (Evans & Greer, 2004). We have combined data from the POES space-138 craft NOAA15 to 19. These data have been corrected from proton contamination using 139 the bow tie method described in Lam et al. (2010). We average all observations in 0.5 140 L^{*} (calculated using the Olson Pfitzer Quiet model (Olson & Pfitzer, 1977)) for direct 141 comparison with the diffusion coefficients from the BAS wave model. As we are using 142 data from multiple POES satellites (NOAA15-19) we have data covering a wide range 143 of magnetic local time sectors but predominately focus between 9-12 MLT for this pa-144 per. For reference the local times of each satellite, for the ascending node, are given in 145 Table 2 of Sandanger et al. (2015). 146

During our calculations of the electron precipitation it is important to know when the T0/T90 detector fields of view (30° wide) are inside/outside the equatorial loss cone (the pitch angle of the loss cone when mapped along the magnetic field to the equator). To do this we project the field of view of the instruments to the equator, using the Olson and Pfitzer (1977) magnetic model.

Figure 1 shows the integral flux measurements made by POES, for three L^* bins 152 of interest between 00-12 MLT during 26-30 March 2013; to show the general trend in 153 the data we have used a line plot however, we note these data are not continuous but 154 rather made from several spacecraft as outlined above. The Kp is shown in the bottom 155 panel. We have used a noise threshold of $1000 \text{ cm}^{-2} \text{sr}^{-1} \text{s}^{-1}$, which the precipitating flux 156 measured by T0 (solid line) is generally below during low Kp. Therefore, when we cal-157 culate the precipitation for this event we are typically looking during moderate to high 158 activity levels. Furthermore, the >300 keV electron flux (not shown) is not above this 159 noise level and hence not considered in this paper. We also note that the modelled lo-160 cation of the plasmapause (blue line in the bottom panel, calculated as described in Meredith 161 et al. (2018)) doesn't go above $L^* = 5$ during this event. 162

There has been some question about the validity of the T0 measurements; Selesnick et al. (2020) suggest the T0 telescope predominately measures stably trapped or quasi trapped flux in the drift loss cone rather than precipitating flux in the bounce loss cone.



Figure 1. Integral flux measurements made by T0 (solid lines) and T90 (dotted lines) for >30 keV (black) and >100 keV (blue) electrons from the POES satellites averaged in 0.5 L^{*} for $5 < L^* < 5.5$ (top panel), $5.5 < L^* < 6$ (second panel), $6 < L^* < 6.5$ (third panel) between 26-30 March 2013. The bottom panel shows the Kp during this event and the blue line demonstrates the modelled location of the plasmapause (dependent on Kp and MLT) from the BAS wave model. The colour of Kp indicates the activity levels with low activity (0 < Kp < 2) shown in green, moderate activity (2 < Kp < 4) in orange and high activity (Kp >4) indicated in red.

However, Rodger et al. (2022) point out that the T0 measurements have been cross-calibrated 166 using multiple different independent data sets (one example being VLF/LF transmit-167 ters by Clilverd et al. (2010)) that do suggest T0 measures the precipitating flux. Fur-168 thermore, we have limited ourselves to measurements above a relatively high noise thresh-169 old $(1000 \text{ cm}^{-2} \text{sr}^{-1} \text{s}^{-1})$, shown by dotted line in Figure 1), where the precipitation should 170 dominate the T0 measurements. 171

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2.2 Geostationary Operational Environmental Satellite (GOES)

We are using data from the MAGED (MAGnetospheric Electron Detector) instru-173 ment on board GOES-15, which provides the differential electron flux at five different 174 energies (40 keV, 75 keV, 150 keV, 275 keV and 475 keV) and has nine telescopes with 175 different look angles (Onsager et al., 1996). It is possible that the flux may vary with 176 pitch angle but by using telescope 9 we are using the closest in pitch angle to T90. We 177 note that the pitch angle for the telescope 9 of MEPED/GOES is changing depending 178 on geomagnetic activity, since the intensity of the ambient geomagnetic field at GOES 179 15 is comparable to the magnitude of geomagnetic field variations, however during this 180 interval the pitch angle is varying approximately between 15° and 10° between 26- 30 181 March 2013 (for reference, T90 has a pitch angle of 3° at GOES L-shells during this in-182 terval). 183

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2.3 Radiation Belt Storm Probe (RBSP-A)

We have obtained data from one of the twin Van Allen Probes (RBSP), Radiation 185 Belt Storm Probes A (RBSP-A) Electric and Magnetic Field Instrument Suite and In-186 tegrated Science (EMFISIS) (Kletzing et al., 2013). The Van Allen Probes have a 9 hour 187 orbit near the magnetic equator with a $\sim 10^{\circ}$ inclination and a perigee of $\sim 1.1 \text{ R}_E$ (Mauk 188 et al., 2013). EMFISIS measures magnetic and electric fields between approximately 10 Hz 189 up to 400 kHz, providing a comprehensive set of magnetospheric wave properties, which 190 are later used to calculate chorus diffusion coefficients. The electron plasma frequency, 191 f_{pe} , is provided as a Level 4 data product and is derived either from the upper hybrid 192 frequency (when visible) or by the lower frequency continuum radiation (Kurth et al., 193 2015a). The electron gyrofrequency, f_{ce} , is found using measurements of the local mag-194 netic field made by the 1s fluxgate magnetometer. 195

We have used data from November 2012, when RBSP were orbiting at high L* on 196 the dawnside, to compare to modelled values of the f_{pe}/f_{ce} used to calculate chorus dif-197 fusion coefficients. We have also used 7 years of RBSP wave and cold plasma measure-198 ments between November 2012 to October 2019. to calculate chorus diffusion coefficients 199 using two different methods, as described later. 200

2.4 Quasi-linear theory 201

As in Reidy et al. (2021), we use the steady state solution to a Fokker Planck equation for pitch angle diffusion from Kennel and Petschek (1966) to calculate the precip-203 itating flux.

$$J_{eq}(E, \alpha_{eq}) = N \ S(E) \ D_{\alpha\alpha}(\alpha_0)^{-1} \left[h(\alpha_0) + \ln\left(\frac{\sin\alpha_{eq}}{\sin\alpha_0}\right) \right],\tag{1}$$

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outside the loss cone $(\alpha_0 \leq \alpha_{eq} \leq \frac{\pi}{2}),$

$$J_{eq}(E, \alpha_{eq}) = N \ S(E) \ D_{\alpha\alpha}(\alpha_0)^{-1} \ h(\alpha_{eq}), \tag{2}$$

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inside the loss cone $(\alpha_{eq} \leq \alpha_0)$ and

$$h(\alpha_{eq}) \equiv \frac{\sqrt{D_{\alpha\alpha}(\alpha_0)\tau}}{\alpha_0} \left[\frac{I_0\left(\frac{\alpha_{eq}}{\sqrt{D_{\alpha\alpha}(\alpha_0)\tau}}\right)}{I_1\left(\frac{\alpha_0}{\sqrt{D_{\alpha\alpha}(\alpha_0)\tau}}\right)} \right].$$
 (3)

 $J_{eq}(E, \alpha_{eq})$ is the equatorial flux distribution for electrons, $D_{\alpha\alpha}(\alpha_0)$ are bounceaveraged pitch angle diffusion coefficients, α_{eq} are the equatorial pitch angles, E is the energy, τ the escape time (assumed to be a quarter of a bounce period), I_0 and I_1 are modified Bessel functions and N is a normalisation factor, S(E) is the source of particles (N and S(E) are defined based on the source spectrum).

For $D_{\alpha\alpha}(\alpha_0)$, we combine contributions from chorus and Coulomb collisions from 213 the BAS-RBM wave model as in Reidy et al. (2021). These waves are used to calculate 214 the $D_{\alpha\alpha}(\alpha_0)$ using the PADIE (Pitch Angle and Energy Diffusion of Ions and Electrons) 215 code, which calculates fully relativistic pitch angel, energy and mixed diffusion coeffi-216 cients for resonant wave particle interactions as described in Glauert and Horne (2005). 217 The BAS wave model is based on measurements from multiple different satellites which 218 are binned by location and geomagnetic activity e.g. the chorus waves described in Meredith 219 et al. (2020). The effects of hiss waves are not included, as we are looking at L^* outside 220 the plasmaphere, as assumed by our modelled plasmapause location shown in the bot-221 tom panel of Figure 1. Diffusion due to EMIC waves are included but are negligible at 222 the energies we consider. At each time of consideration, the $D_{\alpha\alpha}(\alpha_0)$ is evaluated at the 223 edge of the loss cone based on the L^*/MLT location of the spacecraft and the current 224 geomagnetic activity level. The calculation and specifics of these diffusion coefficients 225 will be discussed in more detail in Section 3.3. 226

227 3 Results

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3.1 The impact of the shape of the source spectrum

Figure 2 shows the MLT location of the GOES-15 and POES satellites between 26-229 30 March 2013. The POES flux data are a combination from NOAA-16, -17 and -19 (which 230 have been individually averaged over 2 minutes and $0.5 L^*$ before being combined), be-231 tween $6 < L^* < 6.5$ for consistency with the GOES flux data at geostationary orbit. The 232 GOES flux data are at 2 minute resolution. To find a conjunction between POES and GOES during this time, we require the spacecraft to be within 0.1 hours of MLT of each 234 other and within an hour of UT. Furthermore, before we use the POES data to calcu-235 late the precipitation, we require the entire T0 field of view to be within the loss cone, 236 the entire T90 field to be outside the loss cone, the flux measured by the >30 keV chan-237 nel to be greater than the flux measured by the >100 keV channel and the flux measured 238 by the >100 keV channel to be greater than the flux measured by the >300 keV chan-239 nel. We also imposed a noise threshold of $1000 \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$ and do not use any mea-240 surements when POES is within the longitudinal range of the South Atlantic Magnetic 241 Anomaly. We find three conjunctions that meet this criteria that will be discussed be-242 low, shown by red asterisks in Figure 2. 243

Figures 3a, b and c show the differential flux measured by GOES telescope 9 at the 244 three conjugate times in black asterisks. We have then fitted a power law (black dashed 245 line) and an exponential (black dot-dashed line) to the GOES data at each time. It can 246 be seen for the first two times (a and b), that the data shows better agreement with the 247 exponential fit whereas the third time (c), the data fits better to the power law fit. This 248 is similar to that previously found by Clilverd et al. (2010) and Whittaker et al. (2013). 249 In Figures 3a, b and c, we also show the source spectra fitted from the POES T90 mea-250 surement assuming different spectral shapes: two kappa distributions with $\kappa = 5$ (as used 251 in Reidy et al. (2021) (solid line)) and $\kappa = 2$ (dotted line), then an exponential fit (dot-252



Figure 2. Showing the data from POES and GOES as a function of MLT during event, shown in black and blue respectively. The red asterisks are times when the criteria for a conjugate observation has been met.



Figure 3. Figures a, b and c show the different fitted differential source spectra for the three conjugate times. The in-situ GOES-15 data are shown by black asterisks, fitted exponential and power law source spectra are shown in black dot-dashed and dashed lines respectively. The POES T90 data fitted to an exponential (dot-dashed), power law (dashed), $\kappa = 5$ (solid line) and $\kappa = 2$ (dotted line) are also shown in different colours for the three times. The corresponding calculated T0 precipitating spectra for each source spectra are shown in Figures d, e and f. The time and MLT of the GOES and POES measurements for each conjunction are provided in the top panel where the date format is YYYYMMDD_HH:mm:ss UT.

²⁵³ dashed) line and a power law fit (dashed lines). We note that the $\kappa = 5$ fit gives the low-²⁵⁴ est flux measurements at 30 keV and at the higher energies but there is a 'turn over' in ²⁵⁵ the middle energies where the $\kappa = 5$ fit has the highest flux, the $\kappa = 2$ fit is similar but ²⁵⁶ provides a higher spectrum, as expected, with the exponential fit almost between the two; ²⁵⁷ the power law fit has the highest flux at the higher energies.

At each conjunction time, we have calculated the corresponding precipitating spec-258 tra using equations 1-3 for each of the six source spectral shapes, shown in Figs 3d-f. Ta-259 ble 1 gives the ratio of the calculated to measured T0 precipitating flux at each time for 260 the >30 keV and >100 keV channels for each of the source terms. We note for the POES 261 fitted source terms, there is very little difference across the four shapes of the source spec-262 tra, with a power law doing best for the >30 keV channel at times 1 and 2 but worse 263 for time 3, which coincidentally was the time that the GOES data were best fit by a power 264 law. The GOES power law source term however, does a good job at reproducing the mea-265 sured >100 keV T0 flux at time 3 (with a ratio of 0.87) but is drastically overproduc-266 ing the >30 keV flux (ratio of 5.58). The precipitating flux calculated using the GOES 267 source terms is generally higher than that calculated from POES, this is likely due to 268 the GOES flux measurements being at a higher pitch angle than POES and hence pro-269 viding a larger magnitude of the source flux. Overall from this table, there is no clear 270 fitted spectra representation of the source spectra that is doing significantly better than 271 the rest for both integral channels for all three times. Furthermore, as seen in figures 3d, 272 e and f, there is very little difference between the different calculated precipitation spec-273 tra for the different source spectral shapes, the biggest difference can be seen at 30 keV, 274 where the GOES-based spectra have the highest flux, followed by the POES power law 275 fit. The lowest precipitating flux at 30 keV is from the $\kappa = 5$ fit, which we note was used 276 in Reidy et al. (2021) for their precipitation calculations. The precipitating flux for all 277 the different source spectra falls off exponentially around 200 keV, showing that the hard-278 ness of the spectrum is making very little difference at the higher energies. 279

	Time 1		Time 2		Time 3	
	>30 keV	>100 keV	>30 keV	>100 keV	>30 keV	>100 keV
GOES PL	1.38	0.09	2.43	1.36	5.58	0.87
GOES exp	1.15	0.16	1.88	2.68	3.92	1.45
$\kappa = 2$	0.43	0.07	0.86	0.55	1.93	0.33
$\kappa = 5$	0.40	0.07	0.78	0.66	1.73	0.39
exp.	0.49	0.06	0.86	0.56	1.98	0.32
P.L.	0.59	0.04	1.00	0.35	2.39	0.20

Table 1.	Ratio of calculated t	o measured T0 flux	x from >30 and	>100 keV	channels for	differ-
ent source	spectra at the three c	onjunction times.				

To investigate why the precipitating flux is dropping off so rapidly at the higher 280 energies, we looked back at the solution to the Fokker-Planck diffusion equation from 281 Kennel and Petschek (1966) given by Equations 1-3. Figure 4 shows the pitch angle dis-282 tribution for 30 keV, 100 keV, 250 keV and 500 keV electrons using a POES power law 283 source term (solid line) and also the POES exponential fit (dashed line) at the time of 284 the first POES-GOES conjunction (previously termed Time 1 where, 11 < MLT < 12, $6 < L^* < 6$, 285 0 < Kp < 2). The angle of the loss cone is indicated by the vertical dotted line and fields 286 of view of the POES T0 and T90 are shown by grey shaded regions. As discussed in Theodoridis 287 and Paolini (1967), the shape of the flux within the loss cone is determined by the strength 288 of the diffusion coefficient; the higher the diffusion rate, the flatter the flux in the loss 289 cone becomes, up to the strong diffusion limit (as shown in Figure 4 of Reidy et al. (2021)). 290 The diffusion coefficient from the BAS model used in the flux calculation for each en-291 ergy level is indicated on Figure 4, which decrease with increasing energy, as expected 292



Figure 4. Figure showing the differential flux calculated from Kennel and Petschek (1966) solution for 30 keV (cyan), 100 keV (blue), 250 keV (purple) and 500 keV (pink) electrons with a source spectrum fitted to a power law (solid line) and an exponential (dashed line) based on POES T90 at 20:38.11 UT on 27 March 2013 (i.e. Time 1 in Table 1) The field of view of POES T0 and T90 projected to the equator are indicated by the light and dark grey shaded boxes respectively.



Figure 5. Figure showing the source spectrum (top row) and corresponding calculated precipitating flux spectrum for three different source spectral shapes that have been fitted to the POES T90 data; (a) κ distribution with $\kappa = 5$, (b) Exponential fit, (c) Power law fit between 09-12 MLT, $5 < L^* < 5.5$. The Pearson's linear correlation coefficient and number of points are indicated for each spectra fit for both the >30 and >100 keV channels.

for chorus driven diffusion (e.g. Meredith et al. (2003)). Figure 4 shows for the lower en-293 ergies (30 keV, 100 keV) we are getting a visible difference in the flux within the field 294 of view of T0 whereas at the highest energy considered (500 keV) the precipitating flux, 295 despite having an almost factor of 10 difference in the source flux (visible in the T90 field 296 of view), is falling off so rapidly in the loss cone, it is outside the pitch angle range mea-297 sured by the T0 detector at this location. At 250 keV there is very little difference in 298 the differential flux for the different source terms, we can see from Figure 3 there is a 299 cross over in the different spectra around this energy. Figure 4 demonstrates that despite the increase in the source flux at the higher energies, the calculated precipitating 301 flux is highly dependent on the strength of the diffusion coefficients and therefore, ac-302 cording to the Kennel and Petschek (1966) solution, simply increasing the source flux 303 at higher energies will not drastically change the precipitating flux predicted to be mea-304 sured by the POES T0 detector. 305

We did a wider test of the different source spectra, using the 26-30 March 2013 event shown in Figure 5; We applied this to all data between $5 < L^* < 5.5$ and 09-12 MLT that are above our noise threshold, (this L*/MLT region was selected for ease of comparison with data shown in Sections 3.2 and 3.3). We have used three different shapes for the source spectrum fitted to the POES T90 observations: (a) a kappa fit with $\kappa = 5$ (b) an exponential fit and (c) a power law fit and shown the corresponding calculated precipitating spectrum for each time during the event underneath.

Figures 5 clearly shows that the different source terms, whilst having a significant 313 effect on the amount of flux at higher energies, particularly the power law fit, have min-314 imal impact on the calculated T0 precipitating spectra shown for the three source spec-315 tral shapes. To further demonstrate this we have included the Pearson's linear correla-316 tion coefficient between the calculated and measured T0 precipitating flux on the pre-317 cipitating spectra graph; these are essentially the same for each source spectra, with the 318 power law giving 0.69 for >30 keV, an improvement of 0.01 compared to the other source 319 terms. As discussed above and shown in Figure 4, this is likely due to the strength of 320 the diffusion coefficients at the higher energies. 321

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3.2 Variability of the cold plasma density

As demonstrated by Figure 4, the strength of the diffusion coefficients have a big 323 impact on the shape of the flux in the loss cone when using the Kennel and Petschek (1966) 324 solution. Therefore, another reason for the underestimate of the >100 keV precipitation 325 in Reidy et al. (2021) could be that the BAS diffusion coefficients are not capturing enough 326 diffusion at higher energies. These diffusion coefficients were calculated as described in 327 Horne et al. (2013) using wave and cold plasma data from seven satellites (Meredith et 328 al., 2020). In this wave data base, the wave parameters are binned by pitch angle, en-329 ergy, L*, MLT, magnetic latitude, frequency and geomagnetic activity. One of the pa-330 rameters that go into the diffusion coefficient calculations, provided from this data base. 331 is the cold plasma density, typically discussed as f_{pe}/f_{ce} . The density is known to in-332 fluence the energy at which resonant wave-particle interactions occur. 333

To investigate how well the BAS wave model is capturing f_{pe}/f_{ce} , we compare in-334 situ data from RBSP A during November 2012 (an interval where the orbit of RBSP was 335 at high L^* on the dawnside; note that this is not possible for the 26-30 March 2013 event 336 as the RBSP were not in the right place), with the f_{pe}/f_{ce} from the BAS wave model 337 that would have been used to calculate the chorus diffusion coefficients (selected at each 338 time based on the RBSP location in L^* and MLT and the activity level), shown in the 339 top two panels of Figure 6 respectively. The local f_{pe} measured by the RBSP (top panel 340 of Figure 6) has been projected to the equator assuming a dipole. As in Meredith et al. 341 (2004), the presence of electron cyclotron harmonics (ECH) in the High Frequency Re-342 ceiver are used to determine if the satellite is outside the plasmapause, indicated at the 343 bottom of the first panel in blue (outside) or red (inside). We only show the f_{pe}/f_{ce} from 344 the BAS wave model when the criteria indicates we are outside the plasmapause as we are interested in chorus waves for this study. It can be seen in general, any time larger 346 values of f_{pe}/f_{ce} are measured, the ECH criteria suggests that the RBSP are inside the 347 plasmapause, though there are a few large values of measured f_{pe}/f_{ce} near the begin-348 ning of the month outside the plasmapause during quieter Kp (bottom panel). The modelled f_{pe}/f_{ce} and the ratio of measured to modelled f_{pe}/f_{ce} (third panel) are given when 350 the ECH criteria suggests we are outside the plasmasphere. The modelled f_{pe}/f_{ce} is gen-351 erally lower than that measured by RBSP during November 2012 with the ratio between 352 the modelled and measured f_{pe}/f_{ce} varying 0.1 - 2.8, with a mean value of 0.8 (red line 353 on panel 3). 354

To quantify the effect the cold plasma density has on chorus diffusion at the loss cone, we have re-calculated the chorus $D_{\alpha\alpha}(\alpha_0)$ on the dawnside side between $5 < L^* < 5.5$ with f_{pe}/f_{ce} divided by and multiplied by 2, shown in Figure 7 for low, moderate and high Kp. For comparison, chorus $D_{\alpha\alpha}$ calculated with the original f_{pe}/f_{ce} from the BAS wave model is shown in the top panel. It is clear from Figure 7 that dividing f_{pe}/f_{ce} by two increases $D_{\alpha\alpha}$ at the loss cone at the higher energies and multiplying f_{pe}/f_{ce} by two, decreases the diffusion at the higher energies.

The top row of Figure 8 shows calculated precipitating spectra for the 26-30 March 2013 event discussed in Section 3.1 and analysed in Reidy et al. (2021) for $5 < L^* < 5.5$



Figure 6. Figure showing the f_{pe}/f_{ce} measured by RBSP-A for $5 < L^* < 5.5$, between 06-08 MLT (top panel), the blue and red shading at the bottom of the graph indicate when the ECG criteria suggest the RBSP-A is outside and inside the plasmapause respectively. The f_{pe}/f_{ce} that would be used in the BAS wave model, found using the Kp value and location of the satellite is shown in the second panel, the ratio of the modelled to measured f_{pe}/f_{ce} in the third panel with the mean indicated by the red line. The bottom panel gives the Kp during November 2012 with the colour indicating activity level (green =low activity (0 < Kp < 2), orange = moderate activity (2 < Kp < 4), and red = high activity, (Kp >4)). The modelled location of the plasmapause (LPP) and the location of the RBSP-A between 06-08 MLT are also shown in the bottom panel by blue and black respectively.



Figure 7. Figure showing MLT verses energy dependence of chorus $D_{\alpha\alpha}$ calculated with the 'original' f_{pe}/f_{ce} from the BAS wave model (top), with f_{pe}/f_{ce} divided by two (middle) and multiplied by 2 (bottom) for low, moderate and high Kp between 06-12 MLT for $5 < L^* < 5.5$.

between 09-12 MLT, for a. chorus $D_{\alpha\alpha}$ with f_{pe}/f_{ce} multiplied by two, b. the original 364 chorus $D_{\alpha\alpha}$ (same as Figure 5a), and c. chorus $D_{\alpha\alpha}$ with f_{pe}/f_{ce} divided by two. These 365 calculations assume a kappa distribution as the source term, with $\kappa = 5$ for a consistent 366 comparison with the earlier Reidy et al. (2021) results. This figure clearly demonstrates 367 that by decreasing the cold plasma density (Figure 8c), the precipitating flux increases 368 at the higher energies and by increasing density (Figure 8a), the flux decreases at the 369 higher energies. The bottom row of Figure 8 shows scatter plots of the measured verses 370 calculated T0 precipitation, with the corresponding Pearson's correlation coefficients for 371 the >30 keV and >100 keV electron channels. These show an improvement for the >100 keV 372 channel when the density is decreased, increasing from 0.25 for the original chorus ma-373 trix to 0.37. There is also improvement in the >30 keV with the decreased density. The 374 precipitation calculated using the chorus $D_{\alpha\alpha}$ with f_{pe}/f_{ce} multiplied by two has lower 375 correlation for both electron energy channels. Lines of best fit are also indicated for the 376 >30 keV and >100 keV channels by black and blue dashed lines respectively. 377

378

3.3 RBSP-determined diffusion coefficients

Our current method to calculate the precipitation flux relies on diffusion coefficients that were generated using averaged wave models and plasma density. In these models measurements from multiple satellites, such as wave power and cold plasma density, have been binned by location and activity level and then averaged before calculating the diffusion coefficients. However, Watt et al. (2019) showed that if you calculate the diffusion coefficients from co-located measurements and then take an average, there is a significant difference in the diffusion coefficients.

Here we present chorus diffusion coefficients that have been calculated from RBSP 386 data using two different methods, first using average values, as has previously been done 387 (e.g. Horne et al. (2013)) and used above, and secondly by using co-located measurements 388 of the wave spectra and f_{pe}/f_{ce} to calculate $D_{\alpha\alpha}$ and then averaging, similar to that pre-389 sented in Ross et al. (2021) for EMIC waves and Wong et al. (2022) for magnetosonic 390 waves. Both methods use a modified version of the PADIE code (Glauert & Horne, 2005) 391 which allows an arbitrary wave power spectral density input rather than Gaussian in-392 puts. We have concentrated on the dawnside between 00-12 MLT, for $5 < L^* < 5.5$ as 393 this is where we have RBSP measurements and chorus scattering is known to occur (e.g. 394 Lam et al. (2010)). We have used the same field line model (Olson & Pfitzer, 1977) used 305 in Reidy et al. (2021) for continuity and the ECH criteria from Meredith et al. (2004) 396 is employed to determine if the satellites are outside the plasmapause. The RBSP cho-397 rus diffusion coefficient matrices are computed by combining RBSP data with a profile 398 for how chorus wave power changes with latitude, derived from the VLF database in Meredith 399 et al. (2018). The magnetic latitude profile enables us to map RBSP measurements to 400 magnetic latitudes between 0 < MLAT < 60 and therefore include the effects of high lat-401 itude chorus in our results. The RBSP diffusion matrices also use a new chorus wave nor-402



Figure 8. Calculated precipitating spectra and corresponding scatter plot of measured verses T0 flux for chorus $D_{\alpha\alpha}$ calculated with: (a) f_{pe}/f_{ce} multiplied by 2, (b) the f_{pe}/f_{ce} currently used to calculate the diffusion coefficients (c) f_{pe}/f_{ce} divided by two for the 26-30 March 2013 event between 09-12 MLT for $5 < L^* < 5.5$. The number of points analysed for each POES energy channel (>30 keV and >100 keV) and the Pearson's linear correlation coefficient is given for each case on the scatter plot as well as the line of best fit for the >30 keV (black) and >100 keV (blue) channels indicated by dashed lines. The x=y line is indicated by a dotted line to help comparison.

mal angle model derived from RBSP data composed of different wave normal angle distributions for different spatial location and f_{pe}/f_{ce} bins.

Figure 9 shows chorus-driven $D_{\alpha\alpha}$ at the edge of the loss cone as a function of MLT 405 and energy between $5 < L^* < 5.5$ for different activity levels. The top row were calcu-406 lated using the wave data base described in Meredith et al. (2020) (used in Reidy et al. 407 (2021)), included here for comparison and are the same as Figure 7a for a wider MLT 408 range. The middle row is using the same method of calculation for the $D_{\alpha\alpha}$ but only us-409 ing RBSP data. The bottom row show $D_{\alpha\alpha}$ calculated using co-located measurements 410 from the RBSPs. The biggest differences in the chorus $D_{\alpha\alpha}(\alpha_0)$, are seen in the change 411 from using the entire wave data base to the RBSP data, with some smaller differences 412 due to changes in the method of calculating the RBSP chorus, especially for low Kp. 413

Figure 10 shows a cut through at 100 keV for the three different methods of cal-414 culating chorus-driven $D_{\alpha\alpha}$ at the loss cone for low, moderate and high Kp. For mod-415 erate activity (i.e. 2 < Kp < 3), all the three methods produce similar $D_{\alpha\alpha}$, with the RBSP 416 chorus using co-located measurements being slightly higher in general. The biggest dif-417 ference can be seen for the low Kp, however due to the flux noise threshold we use for 418 the POES measurements, we do not calculate the precipitation during low Kp (see Fig-419 ure 1) and for high Kp MLT < 4 where the RBSP $D_{\alpha\alpha}(\alpha_0)$ for both methods is signif-420 icantly higher than the $D_{\alpha\alpha}(\alpha_0)$ using the entire wave data base at 100 keV. 421

Figure 11 shows the calculated precipitating spectra (top) and the measured verses 422 the calculated precipitation fluxes (bottom) between 09-12 MLT, $5 < L^* < 5.5$ between 423 24-30 March 2013, and is comparable to Figures 5 and 8. As in Section 3.2, we are us-424 ing the $\kappa = 5$ model for the source spectrum for continuity of comparison. Figures 11a, 425 b and c show the results using the diffusion coefficients calculated using the entire wave data base and then the averaged and co-located RBSP measurements respectively. The 427 precipitating spectra is harder for the RBSP-observation determined chorus $D_{\alpha\alpha}$, which 428 has in turn increased the calculated T0 flux for the >100 keV channel, improving the 429 linear correlation from 0.25 to 0.46 and 0.44 for the averaged and co-located methods 430 respectively. These are much larger increases in the correlation coefficient for the 100 keV 431 channel compared to changing the source spectrum (which made very little difference) 432 and by artificially decreasing the density by 2 (which increased the 100 keV correlation 433 to 0.37). 434

Figure 12 shows the Pearson's linear correlation coefficient between the measured 435 and calculated T0 flux for the three different methods of calculating the chorus diffu-436 sion coefficients for >30 keV electrons (a-c) and >100 keV electrons (d-f) for $5 < L^* < 5.5$ 437 between 0-12 MLT. The correlation is only shown for a confidence level above 80% for 438 the >100 keV channel and above 95% for the >30 keV. For reference, the Pearson's cor-439 relation coefficient for both T0 electron energy channels and each MLT sector are given 440 in Table 2. For all the MLT sectors, except 00-03 MLT for the >100 keV channel, the 441 use of the RBSP-observation determined chorus $D_{\alpha\alpha}$ has increased the correlation for 442 both the >30 keV and >100 keV channels compared to using the all chorus wave data 443 base. For the >30 keV channel, the RBSP co-located chorus $D_{\alpha\alpha}$ produce the best com-444 parison results between the calculation and observation, however the >100 keV compar-445 ison is only better for MLT<6, where the RBSP averaged $D_{\alpha\alpha}$ are best. 446

447 4 Discussion

In this paper we have explored the 'missing' higher energy precipitation in the calculations presented by Reidy et al. (2021). We have investigated the impact of the spectral shape used as the source term with conjugate measurements from GOES-15 as well as the effect of the variability of the cold plasma density and the method of calculation on the strength of the chorus diffusion coefficients at the edge of the loss cone.



Figure 9. MLT- energy distribution for chorus pitch angle diffusion coefficients evaluated at the loss cone which have been calculated using average values from all the chorus wave data presented in Meredith et al. (2020) (top row), using average values measured by RBSP (middle row) and using co-located measurements of the wave spectra and f_{pe}/f_{ce} from RBSP (bottom) during low, moderate and high Kp levels.



Figure 10. Chorus-driven pitch angle diffusion coefficients for electrons at 100 keV for different MLT sectors and low, moderate and high Kp levels. Shown for three different calculations: All chorus $D_{\alpha\alpha}$ (blue), RBSP $D_{\alpha\alpha}$ average calculation (black), RBSP $D_{\alpha\alpha}$ co-located measurements (pink).



Figure 11. The precipitating spectra (top) and corresponding measured verses calculated T0 flux (bottom) between 09-12 MLT, $5 < L^* < 5.5$ between 26-30 March 2013 for chorus diffusion calculated using (a) All the wave data from Meredith et al. (2020), (b) The RBSP data and (c) The RBSP data using co-located rather than average measurements of the wave power and f_{ce}/f_{pe} . The Pearson's correlation coefficients and number of points is shown on the scatter plot for each case, as well as a line of best fit for the >30 keV and >100 keV channels in black and blue dashed lines respectively with the x=y indicated by a dotted line for comparison.



Figure 12. Dial plots between 00-12 MLT with noon at the top and dawn to the right, showing the Pearson's correlation coefficient between the measured and calculated T0 flux for different MLT sectors between $5 < L^* < 5.5$ for the >30 keV channel (top row) and the >100 keV channel (bottom row) using chorus diffusion coefficients calculated in three ways. The correlation is shown for at 95% and 80% confidence levels for the >30 and >100 keV channels respectively.



Figure 13. The Pearson's linear correlation coefficients for the calculated and measured >30 and >100 keV T0 in black crosses and blue triangles respectively where the T0 flux has been calculated using the Chorus diffusion matrix with f_{pe}/f_{ce} times by 2 ("Times 2"), the original matrix used in Reidy et al. (2021) ("Orig."), with f_{pe}/f_{ce} divided by 2 ("Div 2"), using averaged wave measurements solely from RBSP (as opposed to the entire wave data base) ("RBSP av") and lastly using co-located RBSP wave measurements ("RBSP co-loc").

$ 5 < L^* < 5.5$	All chorus $D_{\alpha\alpha}$ >30 keV	>100 keV	Av. RBSP chorus $D_{\alpha\alpha}$ >30 keV	>100 keV	co-located RBSP >30 keV	chorus $D_{\alpha\alpha}$ >100 keV
00-03 MLT	0.61	0.46	0.62	0.34	0.68	0.37
03-06 MLT	0.71	0.11	0.80	0.33	0.83	0.37
06-09 MLT	0.60	0.07	0.56	0.46	0.57	0.35
09-12 MLT	0.68	0.25	0.81	0.46	0.81	0.44

Table 2. Pearson's linear correlation coefficient for the measured to calculated T0 precipitating flux between 24-30 March for $5 < L^* < 5.5$ in three hours of MLT bins on the dawnside for >30 keV and >100 keV electron integral flux channels using the three different methods of calculating chorus $D_{\alpha\alpha}$.

Figure 13 shows the Pearson's linear correlation coefficients between the measured 453 and calculated T0 flux using five different variations of chorus-driven diffusion coefficient, 454 all with a $\kappa = 5$ source term. We see a clear improvement in our results, for both >30 keV 455 (black crosses) and >100 keV (blue triangles) energy channels when we decrease the plasma 456 density used in the calculation of the chorus-driven diffusion caused precipitation. This 457 suggests the density used within the BAS model may be too high; similarly, Longley et 458 al. (2022) used the ratio between the precipitating and trapped flux observed by POES 459 on 17 March 2013 to infer a generally lower plasma density than used in BAS-RBM. The 460 next improvement in correlation values seen in Figure 13 comes from using solely RBSP 461 data (as opposed to the averaging approach employing the entire BAS wave data base) 462 to calculate the diffusion coefficients, almost doubling the correlation coefficient for the 463 higher energy channel from 0.25 to 0.46. We suggest these results may also be explained 464 by the potential overestimate in the plasma density in the entire wave data base due to 465 the inclusion of THEMIS data. THEMIS infers the total electron density using measure-466 ments of the spacecraft potential (from the electric field instrument) and the electron 467 thermal speed (from the electrostatic analyser) (Mozer, 1973; Pedersen et al., 1998). The 468 resulting electron densities are associated with a factor of 2 uncertainty (Li et al., 2010). 469 In contrast it has been found that the EMFISIS/RBSP electron density measurements 470 are more accurate than those determined using spacecraft potential to estimate the den-471 sity, as this approach reduces uncertainties due to the effects of cold electron temper-472 atures (Wygant et al., 2013). Therefore, the density measurements from THEMIS in-473 cluded in the entire wave database could result in an inaccurate/higher plasma density 474 than we are seeing from solely using the RBSP data leading to the better correlation in 475 our results we see from using the RBSP derived diffusion coefficients. Figure 13 also shows 476 we are getting slightly better results using the average method of calculation opposed 477 to using co-located measurements. This is in contrast to Ross et al. (2020, 2021), who 478 found using EMIC $D_{\alpha\alpha}$ calculated with co-located rather than averaged measurements, 479 provided better agreement with modelled data from the BAS-RBM and similarly Wong 480 et al. (2022) found co-located measurements of magnetosonic waves improved their re-481 sults. However, these studies were looking at different pitch angles where perhaps the 482 difference in variability within bins makes a larger difference to the diffusion coefficient 483 calculation. 484

For completeness, Table 3 details the Pearson's linear correlation coefficient for the 485 calculated and measured precipitation in the >30 and >100 keV POES flux channels, 486 between 09-12 MLT and 5 5 < L^{*} < 5.5, for each of the tests we present in this paper, 487 and previously shown in Figures 5, 8, 11. In this table we have also included results cal-488 culated using the different source terms with the two RBSP-observation determined cho-489 rus diffusion coefficients which are not shown. As discussed above, we get the biggest 490 improvement to the results found in Reidy et al. (2021) (top row of Table 3), when we 491 are using chorus $D_{\alpha\alpha}(\alpha_0)$ calculated using averaged RBSP data with either a Power law 492

Chorus $D_{\alpha\alpha}$ matrix	Source term	r_{30}	r_{100}
All Chorus	$\kappa = 5$	0.68	0.25
All Chorus	$\kappa = 2$	0.68	0.25
All Chorus	Exponential	0.68	0.25
All Chorus	Power Law	0.69	0.26
$f_{pe}/f_{ce} \ge 2$	$\kappa = 5$	0.64	0.24
$f_{pe}/f_{ce} \div 2$	$\kappa = 5$	0.74	0.37
Av. RBSP	$\kappa = 5$	0.81	0.46
Av. RBSP	$\kappa = 2$	0.81	0.46
Av. RBSP	Exponential	0.81	0.45
Av. RBSP	Power Law	0.82	0.45
Co. loc. RBSP	$\kappa = 5$	0.81	0.44
Co. loc. RBSP	$\kappa = 2$	0.81	0.43
Co. loc. RBSP	Exponential	0.81	0.43
Co. loc. RBSP	Power Law	0.81	0.43

Table 3. Pearson's linear correlation coefficient between the measured and calculated T0 precipitating fluxes between 24-30 March 2013 for $5 < L^* < 5.5$ between 09-12 MLT. in three hours of MLT bins on the dawnside for >30 keV (r_{30}) and >100 keV (r_{100}) electron integral flux channels using the different source terms and chorus diffusion methods in our calculation. All Chorus refers to the chorus matrix calculated using the entire wave data base, av. RBSP and co. loc. RBSP differentiates between the chorus diffusion matrices calculated using averaged and co-located RBSP measurements.

or Kappa source spectrum; we have increased our correlation between the measured and 493 calculated T0 precipitation in this region from 0.68 to 0.82 for the >30 keV channel and 494 from 0.25 to 0.46 in the >100 keV channel. It is not a straight forward answer which source 495 spectrum is best, with a power law giving the best results for the >30 keV channel and 496 a kappa fit being best for the >100 keV channel. However, we have shown the changes 497 in source spectral shape are minimal compared with which chorus diffusion matrix is ap-498 plied when using the Kennel and Petschek (1966) solution to calculate the T0 precip-499 itation (as presented in this paper); it is known the spectral shape is of great importance 500 when considering the precipitation using other methods/ instruments, as shown for ex-501 ample in Clilverd et al. (2010, 2017). 502

The improved correlation in our results for the >100 keV channel is still consid-503 erably less than the correlation found for the >30 keV channel (0.46 compared to 0.82), 504 therefore we are still likely missing something at the higher energies. In Kurth et al. (2015b) 505 they give details on how the electron density are determined from the plasma wave spec-506 trum measured by EMFSIS. They note limitations in identifying the upper hybrid band 507 during geomagnetically active times, when the electron densities are low; during these 508 times they 'fail to identify any spectral features' and leave a gap in the data set. There-509 fore this could lead to a systematic bias in the RBSP plasma density whereby periods 510 of low plasma density, when the diffusion will be higher, are being excluded. This is be-511 cause the density shifts the resonance energy, whereby lower densities result in more dif-512 fusion at the higher energies, as discussed by Allison et al. (2021) and demonstrated in 513 Figure 7 where decreasing the density has increased the diffusion coefficients at the higher 514 energies and increasing the density increases the diffusion coefficients at the lower en-515 ergies. The inclusion of this lower density data from RBSP could therefore increase the 516 diffusion rates and provide the extra diffusion we are missing at the higher energies, how-517 ever determining such is an extensive piece of work which we leave to future studies. Other 518 avenues to improve our results include using a more dynamic geomagnetic field model, 519 such as TS04 (Tsyganenko & Sitnov, 2005) (rather than Olson Ptizer Quiet model which 520

is for quiet geomagnetic times), to calculate L^* and our diffusion coefficients, particu-521 larly considering our results are primarily from periods with moderate to high Kp. Fur-522 thermore, as stated in Reidy et al. (2021), we are using averaged rather than event-specific 523 diffusion coefficients to calculate precipitation during an event and therefore analysing 524 over more events may provide us with a bigger picture and improve our results. Lastly, 525 it is possible that highly non-linear effects, which are not included in quasi-linear the-526 ory, could enhance the diffusion and increase the precipitation. 527

5 Conclusion 528

In this study we have have improved on the calculated T0 precipitating fluxes pre-529 sented earlier in Reidy et al. (2021), on the dawnside particularly for the higher ener-530 531 gies by using $D_{\alpha\alpha}$ calculated from RBSP measurements. We have investigated the method of calculation by experimenting with different source spectral shapes, as well as differ-532 ent versions of the BAS chorus-driven diffusion matrix. The key results of this paper can 533 be summarised as follows: 534

- Using our current method of calculation (the Kennel and Petschek (1966) solu-535 tion to the diffusion equation), increasing the hardness of the source spectrum has 536 a minimal effect on the amount of calculated T0 precipitation. 537 • We have demonstrated that using chorus diffusion coefficients that have been cal-538 culated assuming a lower cold plasma density (f_{pe}/f_{ce}) divided by 2) significantly 539 increases the precipitation at higher energies, towards the magnitudes which are 540 closer to those observed. This is because reducing the cold plasma density increases 541 the diffusion rates at higher energies and results in more particles being precip-542 itated.
- We have found that using chorus $D_{\alpha\alpha}(\alpha_0)$ calculated with RBSP data improves 544 our results compared to chorus $D_{\alpha\alpha}(\alpha_0)$ calculated from data compiled from many 545 satellites (presented in Meredith et al. (2020)). This is most likely due to the more 546 accurate wave measurements from RBSP than other spacecraft included in the whole 547 BAS model (e.g. THEMIS). 548
- We still find there is a better correlation between the calculations with the POES 549 T0 >30 keV electron channel measurements compared to that for the >100 keV 550 channel, suggesting there is still some missing diffusion at the higher energies. 551

6 Open Research 552

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The POES particle data used in this study came from NOAA National Geophys-553 ical Data Centre for the (https://ngdc.noaa.gov/stp/satellite/poes/dataaccess 554 .html). The Kp indices were downloaded from the OMNI database (https://omniweb 555 .gsfc.nasa.gov/). The Chorus wave pitch angle diffusion coefficients calculated for use 556 in this study have been published in the Polar Data Centre (https://doi.org/10.5285/ 557 5ef0d6cd-67c2-48fc-8a6a-dfe44a63979e) (Reidy et al., 2023). 558

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Figure 1.



Figure 2.



Figure 3.



Figure 4.


Figure 5.



Figure 6.



Figure 7.



Figure 8.



Figure 9.



Figure 10.

 $5.0 < L^* < 5.5$ E = 100 keV



Figure 11.



Figure 12.



Figure 13.

