

1 Techniques to Determine the Quiet Day
2 Curve for a Long-Period of Subionospheric
3 VLF Observations

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4 Very low frequency (VLF) transmissions propagating between the conduct-
5 ing Earth's surface and lower edge of the ionosphere have been used for decades
6 to study the effect of space weather events on the upper atmosphere. The
7 VLF response to these events can only be quantified by comparison of the
8 observed signal to the estimated quiet-time or undisturbed signal levels, known
9 as the quiet day curve (QDC). A common QDC calculation approach for pe-
10 riods of investigation of up to several weeks is to use observations made on
11 quiet days close to the days of interest. This approach is invalid when con-
12 ditions are not quiet around the days of interest. Longer term QDCs have
13 also been created from specifically identified quiet days within the period and
14 knowledge of propagation characteristics. This approach is time consuming,
15 and can be subjective. We present three algorithmic techniques, which are
16 based on either 1) a mean of previous days' observations, 2) Principal Com-
17 ponent Analysis, or 3) the Fast Fourier Transform (FFT), to calculate the
18 QDC for a long-period VLF dataset without identification of specific quiet
19 days as a basis. We demonstrate the effectiveness of the techniques at iden-
20 tifying the true QDCs of synthetic datasets created to mimic patterns seen
21 in actual VLF data including responses to space weather events. We find the
22 most successful technique is to use a smoothing method, developed within
23 the study, on the dataset and then use the developed FFT algorithm. This
24 technique is then applied to multi-year datasets of actual VLF observations.

1. Introduction

25 Man-made VLF transmissions propagate for long distances with low attenuation in
26 the Earth-Ionosphere waveguide between the conductive Earth surface and the lower
27 edge of the ionosphere (D-region). This manner of propagation is termed subiono-
28 spheric. The radiation propagates in a modal fashion within the waveguide, along
29 the great circle path between transmitter and receiver, with the received amplitude
30 and phase of the transmissions being a superposition of these modes [*Wait*, 1996a;
31 *Lynne*, 2010]. Variations in the waveguide over time change the modal mix, causing
32 variations in the observed signal amplitude at a receiver. The primary, temporally
33 varying parameter of the waveguide is the D-region reflection height, which varies on
34 a regular diurnal basis with the presence of solar radiation as well as irregularly in
35 response to space weather events. For an overview of historical VLF science see *Barr*
36 *et al.* [2000].

37 Diurnal variations in VLF observations have been used to determine the relation-
38 ship of solar zenith angle to D-region parameters for the day-time ionosphere [*Thom-*
39 *son*, 1993], and determine similar parameters for the night-time ionosphere [*Thomson*
40 *et al.*, 2007], when solar radiation has a less dominant influence. The most dramatic
41 modal variations occur as the day-night terminator passes across the transmitter-
42 receiver path, with the varying modal superposition causing very deep minima in
43 signal amplitudes. *Clilverd et al.* [1999] related the timing of a set of twilight modal
44 minima to the moving location of the terminator along a path at a height of 75 km.
45 Lightning sferics within the VLF frequency band have also been used to determine

46 day and night-time D-region parameters (e.g., [*Han and Cummer, 2010a, b; Shao*
47 *et al., 2013*]).

48 Space weather events, e.g., solar flares, energetic electron precipitation (EEP) from
49 the radiation belts, and solar proton events (SPEs), can significantly increase the
50 ionization rate in the upper atmosphere, which leads to a reduction in the D-region
51 height and thus a perturbation in the received VLF amplitude [*Clilverd et al., 2009*].
52 To study the effect of space weather events on the ionosphere using VLF propagation
53 it is important to have a method of determining the undisturbed diurnal variation
54 in the VLF observations, known as the Quiet Day Curve (QDC). Once the QDC
55 is determined the perturbations caused to the VLF signals by these events can be
56 quantified.

57 In this paper we report on the development of three algorithmic QDC finding
58 techniques for long-period subionospheric VLF datasets. The first technique is based
59 on the QDC technique from *Simon Wedlund et al. [2014]*, who created their QDC from
60 the combined curve of several quiet days prior to a period of geomagnetic disturbance
61 that they were investigating. The second technique follows the QDC finding technique
62 based on Principal Component Analysis reported by *Collier [2009]* and *Wautelet*
63 *and Warnant [2012]*. These studies selected the principal components accounting
64 for the most variance in their datasets for transforming back to data space. The
65 third technique uses a 2-dimensional Fast Fourier Transform to identify the discrete
66 spectrum of the dataset before applying restrictions developed within this study to
67 the spectrum and inverting the transform to provide the QDC. These techniques all

68 result in QDCs that reproduce both the diurnal and annual amplitude variations in
69 the datasets, to some extent. A two-step pre-smoothing method, which was developed
70 to improve the QDC technique results, is also described. Each technique is evaluated
71 by its success at identifying the true QDC of synthetic datasets, which were created
72 to mimic the behavior of real VLF datasets. The best technique is then applied to
73 datasets of real amplitude observations of subionospherically propagated VLF and
74 the results shown for example events. The development of a successful QDC finding
75 algorithm for long-term datasets will allow for the detection of and statistical analysis
76 of the ionospheric response to space weather events, observed through subionospheric
77 VLF.

2. Datasets

2.1. AARDDVARK VLF Observations

78 The Antarctic-Arctic Radiation-belt (Dynamic) Deposition-VLF Atmospheric Re-
79 search Konsortium (AARDDVARK) [*Cilverd et al.*, 2009] is a global network of VLF
80 receivers located primarily in the polar regions [[http://www.physics.otago.ac.nz/
81 space/AARDDVARK_homepage.htm](http://www.physics.otago.ac.nz/space/AARDDVARK_homepage.htm)]. The AARDDVARK receivers providing data
82 for this study are located near Scott Base (SB: 77°50'S, 16°39'E), Antarctica and
83 Edmonton (EDM: 53°21'N, 112°58'W), Canada. The Scott Base receiver provides
84 examples of long-distance transmitter-receiver great circle paths, while the Edmon-
85 ton receiver provides examples of relatively short-distance paths. Observed signals
86 from short and long paths are expected to behave differently due to the attenuation
87 of higher level modes with distance [*Wait*, 1996b]. The datasets chosen from each re-

88 ceiver have the strongest signals of those monitored by those receivers and so provide
89 relatively clear examples. Both receiver installations use UltraMSK software [*Clilverd*
90 *et al.*, 2009] to process the VLF observations into amplitude and phase data. Fig-
91 ure 1a shows the locations of the two receivers, the monitored VLF communications
92 transmitters that we utilize in our study, and the great circle paths between them.

93 In the current study we use only the VLF amplitude data, because there are ex-
94 tra difficulties in making long-term phase datasets consistent. An example is unpre-
95 dictable phase jumps across periods when the transmitters are turned off, as discussed
96 in *Rodger et al.* [2012]. We reduce the 0.2 s resolution raw data to 1 minute resolution
97 by averaging, i.e., each data point in the reduced dataset is obtained by calculating
98 the median of the 300 data points in that minute of raw data. This averaging removes
99 much of the noisy variation inherent in the received signal from modulation of the
100 transmission. By the law of large numbers, over the course of 1 minute this variation
101 has a Gaussian distribution.

102 For the techniques described in this paper the data was arranged into a matrix such
103 that each row contained one day's worth of data (1440 data points), with the rows
104 ordered sequentially in time. Periods of abnormal transmission or interference from
105 the receiver surroundings were removed from the dataset. Abnormal transmission
106 periods were identified heuristically as when the signal dropped suddenly to the
107 noise floor between 1 minute and the next and then later returned to normal signal
108 levels. Periods of noise interference from the receiver surroundings were identified by

109 comparison to observations on the 23.0 kHz frequency, which is rarely transmitted
110 on.

111 Consistent temporal spacing of data points is essential for our QDC finding tech-
112 niques, so periods when data was missing or removed were included as approximating
113 values. The data approximations were done by combining the ‘linear’ method of the
114 ‘interp1q’ MATLAB function for data gaps of >2 hr duration, the ‘TriScatteredIn-
115 terp’ 2-dimensional surface interpolation function (using the dimensions of the data
116 matrix) for data gaps 2 hr–2 days, and a median of the same numbered days of data
117 from surrounding years for longer data gaps to maintain the overall coherence of the
118 diurnal pattern within the approximated values.

119 Figure 1b shows 32 months of amplitude observations of the NDK (25.2 kHz) trans-
120 mission received at the AARDDVARK antenna located near Edmonton, Canada. The
121 great circle path between transmitter and receiver is completely dark in the primar-
122 ily red region between 02 and 12 UT, and fully Sun-lit in the green to orange region
123 between 14 and 24 UT. The border between the night and day regions is defined by
124 the twilight modal minima, which vary their time of occurrence regularly through
125 each year according to when the day-night terminator crosses the path.

2.2. Synthetic Dataset Creation

126 We created synthetic amplitude datasets for the purpose of evaluating the success
127 of our QDC finding techniques at identifying the true underlying QDC of a dataset.
128 These synthetic datasets were designed to be representative in their general response
129 to light levels along a propagation path and to space weather events, rather than

130 be a true model of the VLF dataset for the equivalent path. The synthetic dataset
 131 matrices contain four years of data at one minute resolution. Like the AARDDVARK
 132 data matrices, each row is one day of data arranged in UT time. Background patterns
 133 in the synthetic amplitude data simulate the general patterns seen in VLF amplitude
 134 data, with periods designated day-time (path fully Sun-lit), night-time (path fully
 135 dark) and twilight-time (day-night terminator along the path).

136 We present one of our synthetic datasets here, shown in Figure 1c, to illustrate
 137 the approach. Figure 1b shows AARDDVARK observations for the equivalent path:
 138 NDK-EDM (shown in Figure 1a). The day, night, and twilight-times of the synthetic
 139 dataset are defined by the solar zenith angles (SZA) at NDK and EDM. The diurnal
 140 variation in the synthetic dataset consists of four sections; a constant-valued section
 141 representing the VLF response to night-time conditions (approximately 02–12 UT), a
 142 curved section representing the VLF response to day-time conditions (approximately
 143 14–24 UT), and two twilight sections separating the night and day-times, each with
 144 a single sinusoidal minima representing the twilight modal minima as seen in VLF
 145 data. The day-time curve ($Data_{day}$) is calculated as

$$146 \quad Data_{day} = -(SZA_{NDK} + SZA_{EDM})/2 + 90$$

147 where SZA_{NDK} and SZA_{EDM} are the SZAs at NDK and EDM, respectively. A
 148 long-term trend of a single sinusoidal cycle is imposed on each column in the matrix.
 149 The diurnal variation is added to the long-term trend to form the background of the
 150 synthetic dataset, which is the true QDC that our techniques are aiming to identify.
 151 This background forms the dominant variation seen in Figure 1c. Perturbations are

152 imposed, by addition, on the synthetic background to represent the VLF response to
 153 solar flares, EEP, and multi-day disturbances to the D-region. A fourth component
 154 imposed on the background represents the effect of random noise on the VLF signal.
 155 Figure 1d shows the background and combined data for a representative day from
 156 the synthetic dataset.

157 Across the 4 years of our synthetic dataset we impose 5000 “EEP events”, which
 158 we represent by downward pointing triangles, and 1000 “solar flare” events. The
 159 equation used to represent a solar flare event (*Flare*) is

$$160 \quad Flare = 2x \exp(-x/size)$$

161 where x is minutes from the start of the event, and $size$ is a random scale factor from
 162 1 to 20, but biased to the lower end of the range. The imposed EEP events are placed
 163 only in the night-time region of the dataset, while the solar flare events are placed only
 164 in the day-time region of the dataset. The timing of both the EEP and the solar flare
 165 events is otherwise random, but biased towards periods of geomagnetic disturbance.
 166 While these events may not be strictly linked to geomagnetic disturbance, this bias
 167 gives a good representation of the clustering of space weather events which occurs
 168 in the “real world”. K_p index values for the four years spanning January 2009 to
 169 December 2012 (sourced from <http://wdc.kugi.kyoto-u.ac.jp>) provide a simple proxy
 170 for both solar and auroral activity and are used to supply the bias, where a higher
 171 K_p will lead to more imposed synthetic EEP and solar flare perturbations. The
 172 magnitude of the imposed EEP and solar flare events is randomly generated within

173 the range 0.6–15 dB, which is representative of the range of responses caused by solar
174 flares and EEP seen in real VLF datasets.

175 Multi-day perturbations are included to simulate the effect of longer space weather
176 events, such as SPEs, or longer-term geomagnetic disturbances. In our synthetic
177 datasets the timing and strengths of these perturbations in the dataset are determined
178 by the D_{st} index values (sourced from <http://wdc.kugi.kyoto-u.ac.jp>) for the same
179 4 year period as used for the K_p -based perturbations. The range of D_{st} values in the
180 period was divided into disturbance levels, which were used to assign perturbation
181 values to each entry in the synthetic data matrix. These added values were smoothed
182 to remove sharp steps from the perturbations. The magnitude range of the added
183 values is 0–5 dB, negative during night-time and positive during day-time. We placed
184 no restrictions on the length of the multi-day perturbations, beyond those inherent
185 in the D_{st} dataset disturbance levels.

186 The added noise component consists of random values selected from a zero-centered
187 Gaussian distribution in the range $\pm x$ that are added to each data point in the day-
188 time and night-time sections of the dataset. We define x from the uncertainties
189 reported by *Rodger et al.* [2007]. The distribution standard deviation during day-
190 time is 0.02 to give an x of 0.1 dB and during night-time is 0.1 to give an x of
191 0.5 dB.

3. Technique Descriptions

192 Below we give descriptions of the QDC finding techniques developed in this study.
193 We also describe the pre-processing addition that we developed to improve the results
194 of the techniques. In the development of these algorithms, we have aimed to make
195 them generic and not specific to one known dataset. As such, these approaches should
196 be valid for any subionospheric VLF amplitude dataset of sufficient duration.

3.1. Combined Daily Curve

197 This technique generalizes the method used by *Simon Wedlund et al.* [2014]. They
198 calculated their QDC from the combined curve of several identified quiet days of VLF
199 amplitude observations that occurred shortly before a period of geomagnetic distur-
200 bance. In the current study this method is generalized by applying the technique
201 with no regard for the level of disturbance in the previous days' data, i.e., there is no
202 attempt to determine if the previous days are indeed quiet. This is done so that our
203 technique does not rely on the time-consuming manual identification of quiet days
204 within a dataset. We therefore note that the calculated QDC will be of lower quality
205 than if we knew the utilized observations came from a truly quiet period. Thus, this
206 technique may best suit periods of lower solar activity. We refer to this method as a
207 Combined Daily Curve (CDC).

208 The CDC is created by averaging data from the 3 days prior to the day of interest.
209 The CDC technique assumes that the diurnal pattern in VLF data changes very little
210 from day to day, except in response to ionospheric perturbations, which the averaging

211 is expected to remove. This assumption is based on examination of diurnal patterns
 212 in VLF datasets (e.g., the relatively regular variations seen in Figure 1b).

213 The CDC is calculated at 10 minute resolution, with each value in the CDC being
 214 averaged from the same respective 10 data points in each of the previous three days.
 215 Thus, each average value is calculated from thirty 1 minute data points to match the
 216 thirty data points, from a single day, that were used by *Simon Wedlund et al.* [2014]
 217 for their QDC value calculation. The CDC is then interpolated back to 1 minute
 218 resolution, using the MATLAB function 'interp1' with the 'linear' method, for direct
 219 comparison with the data. We change the resolution in this manner to reduce the
 220 influence of any one data point on the result. The first 3 days of data in the matrix
 221 do not have corresponding CDCs as they do not have at least 3 days prior to them.

3.2. Principal Component Analysis

222 Principal Component Analysis (PCA) is a tool used in multivariate analysis to
 223 expand a dataset along its directions of maximal variance. For analysis of data vari-
 224 ation, this expansion is sufficient. However, it is possible to summarize the patterns
 225 in a dataset by selecting expansions along a limited number of directions of highest
 226 variance and recombine them [*Collier, 2009*]. For the purpose of this QDC finding
 227 technique, we assume that the majority of the variance in the dataset comes from
 228 the regular diurnal patterns of the data and is thus concentrated in the lower ordered
 229 PCA directions.

230 The steps of the PCA QDC finding technique for an $m \times n$ data matrix \mathbf{X} are as
 231 follows.

232 1. Create the re-centering matrix $\bar{\mathbf{x}}$, which has the entries of each column as the
 233 mean of the corresponding column of \mathbf{X} .

234 2. Calculate the covariance matrix \mathbf{S} , of the recentered data matrix.

$$235 \quad \mathbf{S} = \frac{1}{m-1}(\mathbf{X} - \bar{\mathbf{x}})'(\mathbf{X} - \bar{\mathbf{x}}).$$

236 3. Find the eigenvectors and eigenvalues of \mathbf{S} . These should be sorted in decreasing
 237 order by the eigenvalues. The eigenvectors are the directions of maximal variance for
 238 the PCA process and the corresponding eigenvalues give the variance accounted for
 239 by each direction.

240 4. Project the recentered data matrix onto the eigenvectors of \mathbf{S} to find the princi-
 241 pal components (**PCs**). Defining \mathbf{G} as the matrix of eigenvectors, arranged column-
 242 wise, the matrix of principal components \mathbf{Y} , is

$$243 \quad \mathbf{Y} = (\mathbf{X} - \bar{\mathbf{x}})\mathbf{G}.$$

244 Each column of \mathbf{Y} is a single PC. The PCs are ordered according to the variance
 245 accounted for by their corresponding directions, with the first being the projection
 246 of the recentered data matrix onto the direction of highest variance.

247 5. Choose and apply the criteria to be used for limiting the number of PCs. We
 248 use the Kaiser criterion [*Kaiser*, 1960], which retains only those PCs that individually
 249 account for more than the mean variance over all the PCs.

250 6. Invert the projection for all retained PCs, sum them together and add the re-
 251 centering matrix. With $\mathbf{y}_{(1,2,\dots,i)}$ and $\mathbf{g}_{(1,2,\dots,i)}$ defined as containing the retained PCs

252 and corresponding eigenvectors respectively, the resulting QDC matrix \mathbf{Q}_{PCA} , is

$$253 \quad \mathbf{Q}_{PCA} = \mathbf{y}_{(1,2,\dots,i)} \mathbf{g}'_{(1,2,\dots,i)} + \bar{\mathbf{x}},$$

3.3. Fast Fourier Transform

254 The Fast Fourier Transform (FFT) is used to identify the discrete frequency spec-
 255 trum of a digital dataset. In this study the two dimensions of the FFT are the diurnal
 256 variation in the rows of the data matrix and the day-to-day variation, which includes
 257 the yearly variation, in the columns of the data matrix. Our FFT QDC finding
 258 technique uses the 2-dimensional transform to calculate the spectrum of a dataset,
 259 which is then restricted as described below. We calculate the inverse transform of
 260 the restricted spectrum to provide our QDC. *Amidror* [2013] gives an overview of
 261 the transform in multiple dimensions including details of various issues to be aware
 262 of when using the transform.

263 In this technique we want to remove as much of the perturbation contribution from
 264 the spectrum as possible while retaining as much of the background contribution as
 265 possible, as this represents the true QDC we are trying to find. The central aspect
 266 of this technique is the identification of the spectral components that are dominated
 267 by the perturbation spectrum. Once these unwanted components are identified, we
 268 remove their contribution to the spectrum by setting them to zero. The QDC is
 269 taken as the real component of the resulting matrix from the inverse FFT. Note that
 270 providing the spectrum restrictions maintain the symmetry properties of the original
 271 spectrum, the result of the inverse FFT will have no imaginary component.

272 The linear property of the FFT allows for the examination of the features of the
273 synthetic background spectrum independently from the perturbation spectrum. From
274 this examination we are able to identify consistent features of these spectra across
275 multiple synthetic datasets with different backgrounds and thus develop methods to
276 identify perturbation-dominated spectral components for removal from the spectra.

277 The first spectral restriction is the removal of certain rows of the FFT spectrum
278 to clarify the yearly, including seasonal, variation of the dataset. For this clarifica-
279 tion to be most effective, the dataset is required to be a whole number of years, say
280 p , in length. Cutting the dataset prior to application of the FFT may be required
281 to achieve this. The yearly background pattern of a p -years length dataset repeats
282 p times in the vertical direction of the data matrix. This regular repetition places
283 the background-related spectral components on the p^{th} -multiple vertical frequencies,
284 or rows from the center, of the spectrum. Spectral leakage is a frequency smearing
285 artifact in the FFT that results from the effective discrete truncation of a continuous
286 function [Amidror, 2013]. It causes all spectral components in the spectrum to con-
287 tribute to those surrounding them, in this case the result is that the non- p -multiple
288 rows of the spectrum have some contribution from the background patterns. By lim-
289 iting the dataset to whole numbers of years we minimize that contribution, allowing
290 us to assume that the non- p -multiple rows are perturbation-dominated. Thus, by
291 keeping the dataset to p years, we can immediately identify the non- p -multiple rows
292 of the spectrum as being perturbation-dominated and set their components to zero
293 for QDC generation.

294 This first spectral restriction essentially requires datasets to be of longer duration
295 than two years to allow for row removal in the spectrum. Due to this requirement,
296 our FFT QDC finding technique is not valid for VLF datasets shorter than 2 years.

297 The second spectral restriction is the removal of two regions of the spectrum matrix
298 that are consistently perturbation-dominated and are located vertically up and down
299 from the center of the matrix, and the retention of background-dominated regions.
300 Separate examination of background and perturbation spectra from our synthetic
301 datasets showed us the regions in the combined spectra where each would be ex-
302 pected to be dominant. The strong spectral components of the background layers
303 are located in the center of their spectra, fanning outwards horizontally and diago-
304 nally with decreasing magnitudes in patterns specific to each background. Figure 2a
305 shows the spectral magnitudes of the central section of the synthetic spectrum. Here
306 the background-related pattern is seen on every 4th row as a higher magnitude than
307 surrounding values. None of the background spectra fan out in the vertical direc-
308 tions. The strongest spectral components of the perturbation layers are located in
309 the central column of their spectra (the vertical green columnar region in Figure 2a),
310 symmetrically reducing in magnitude with horizontal distance. From these observa-
311 tions we find that the two triangular regions located in the vertical directions from the
312 center of the spectrum have little contribution from the background spectra and are
313 thus perturbation-dominated. The boundaries of the region of strong background-
314 related spectral components are different for each background and must be identified
315 separately for each dataset. Once the boundaries of the region of significant back-

316 ground contribution are identified, the spectral components in the triangular regions
317 outside of the boundaries are easily set to zero using a stencil.

318 The third spectral restriction is the removal of low-energy spectral components in
319 the high frequency regions of the spectrum matrix. At the edge of the spectrum
320 matrix, where the frequencies are highest, the spectral components are perturbation-
321 dominated and the spectral magnitudes are relatively low. It is necessary to identify
322 the border of the matrix region within which the background-related spectral compo-
323 nents are dominant. This is the point where the distinct pattern of the background-
324 dominated spectral components is subsumed into the general spectrum. A spectral
325 energy limit is employed, with the limit chosen as the lowest energy at which the
326 background pattern is retained and a minimum of spectral components from outside
327 of the pattern are included. This method is less subjective than a determination
328 through visual inspection to find the border of the background-dominated region of
329 the spectrum. The spectral energy limit is different for each spectrum due to the
330 differing background patterns in each corresponding dataset. For the chosen energy
331 limit, a plot of the inverse FFT of the discarded spectral components should not
332 include background patterns from the dataset or periodic variations of greater than
333 0.1 dB magnitude.

334 The first spectral restriction tends to remove contributions from long-term trends
335 to the spectrum of the dataset, due to VLF dataset long-term trends likely being a
336 response to the solar activity cycle of 11 and 22 years. Unless the dataset is itself
337 a multiple of 11 years in length, the main trend-related components are lost at the

338 row removal stage. Thus for this QDC finding technique to take into account any
339 long-term trends, an extra step is needed to re-include the strongest of the removed
340 spectral components in the low frequency region of the matrix to the spectrum prior
341 to the inverse transform.

342 The final synthetic spectrum, after all the restrictions have been applied, is shown in
343 Figure 2b. As with Figure 2a we show only the spectral magnitudes from the central
344 section of the spectrum. The combination of the row removal and stencil restrictions
345 has removed the visible contribution of the perturbation-dominated components in
346 the central region of the spectrum, while the removal of lower energy components
347 shows the border of the background-dominated matrix region.

3.4. Additional Smoothing

348 As will be reported in Section 4.2, the three basic QDC finding techniques, de-
349 scribed above, produce promising results when applied to our synthetic datasets. We
350 also investigated methods to pre-smooth the datasets with the aim of improving the
351 results from the basic techniques. We found that a two-step pre-processing approach,
352 which involves the removal of the most disturbed days of data and then a smoothing
353 of the resulting matrix, applied to the dataset prior to application of the QDC finding
354 technique provided an improvement in the results for the day-time and night-time
355 regions of the matrix. These pre-processing methods are described below.

356 The results of all three QDC finding techniques are negatively influenced by periods
357 of significant disturbance in the datasets to some degree. We investigated nearest
358 neighbor distances [*Cover and Hart, 1967*] as a method of defining the disturbance

359 level of a row of data. Figure 3a shows the nearest neighbor distance for each row of
360 the synthetic perturbation matrix plotted against the nearest neighbor distance for
361 the corresponding rows of the full synthetic dataset. Here we see that rows with higher
362 dataset distances also have higher perturbation distances. From this relationship, we
363 determine that the dataset nearest neighbor distance of a row is a good indicator
364 for the actual disturbance level of a row. We therefore remove from the data matrix
365 those rows with the highest 10 % nearest neighbor distances, as the most disturbed.
366 In Figure 3a this limit is marked by a dashed vertical line.

367 We then smooth the data, which serves two purposes: to replace the re-
368 moved data from disturbed days and reduce the influence of short term perturba-
369 tions, i.e., solar flares, on the QDC. We use the ‘rloess’ method of the ‘smooth’
370 function from the MATLAB® software package’s Curve Fitting Toolbox. This
371 method is a “local regression using weighted linear least squares and a 2nd de-
372 gree polynomial model” that “assigns lower weight to outliers in the regression”
373 [www.mathworks.com/help/curvefit/smooth.html]. The ‘rloess’ method was pre-
374 ferred for the smoothing over a moving average, because of the lower influence of
375 outlying values on the result under this method. This smoothing method fills gaps
376 in the input data as part of the algorithm. We found that smoothing over the gaps
377 from the removed disturbed days in the data matrix improves our results even more
378 than filling them with representative values. The long-term trend in the data is not
379 significantly affected by this method of smoothing as shown by the daily data means
380 presented in Figure 3b. The smoothed dataset daily means show significantly less

381 variation than those of the unsmoothed, full, dataset while also remaining close to
382 the background daily means.

383 The smoothing is done both column-wise and row-wise in the data matrix. The
384 column-wise smoothing is intended to remove single day perturbations, which can
385 be considered outliers within the general shape of the data from day-to-day, and
386 mitigate the effect of multi-day perturbations, such as SPEs. The row-wise smoothing
387 is intended to further reduce the effect of noise around the signal.

388 Care must be taken in choosing the span for the smoothing. Too high a span and
389 the desired background patterns in the data are lost, too low and the smoothing is
390 practically pointless. We tested a range of spans on various of our synthetic datasets
391 to determine the level required under these constraints. For the twilight-times, we
392 found that a span of 7 data points provides adequate smoothing of perturbations
393 without significantly altering the shape of the minima. A higher span is possible for
394 the day-time and night-time regions of the data matrix. We found that a span of
395 13 data points provided very good smoothing while limiting the addition of negative
396 artifacts to the smoothed data matrix in these regions. We therefore smooth the
397 data matrix twice, once at a span of 7 and once at 13, and combine the twilight-time
398 region of the 7-span result with the day and night-time regions of the 13-span result
399 to give our final smoothed dataset for application of a QDC finding technique.

400 A low-pass filter might be used here as an alternative to the smoothing. However,
401 it is not clear whether this style of filter would provide a significant enough improve-
402 ment to the results of the method described above to justify the added subjectivity

403 of determining the cut-off frequency for each dataset. Our smoothing method is con-
 404 venient to the MATLAB® user and requires little subjectivity in the identification
 405 of the required span, which can then be easily translated across different datasets.

4. Testing Techniques on Synthetic Data

4.1. Method to Quantify Technique Success

406 We evaluate the success of our QDC techniques by calculating a parameter to
 407 indicate how close our QDC matrices are to the synthetic background, which is the
 408 true QDC of the synthetic dataset. This parameter allows us to directly compare the
 409 success of our techniques. We calculate this parameter from the difference between
 410 the QDC and the background, which we refer to as the Comparison. Clearly, it
 411 is only possible to determine this parameter for synthetic datasets due to the true
 412 background being unknown for real VLF observations.

413 Our indicative parameter is based on the L^2 vector norm and so we will refer to it
 414 as the norm for the remainder of this study. The equation used to define the norm is

$$415 \quad \|\mathbf{v}\| = \sqrt{\sum_i v_i^2/n}$$

416 where $\|\mathbf{v}\|$ is the norm, v_i are the entries in the relevant section of the Comparison
 417 matrix and n is the number of entries in the section. The norm parameter is higher
 418 than a simple average of absolute values due to the squaring of the entries. It has no
 419 direct physical meaning, being used here as an estimation of the outer variability of
 420 the Comparison matrix. The norm can be calculated for each section of the Com-

421 parison matrix, night-time, day-time, and twilight-time, as well as for the complete
422 matrix. This allows us to compare technique success between Comparison sections.

423 For our technique evaluation we use ten different synthetic datasets, with identical
424 backgrounds, that differ only in the random timing and magnitude of the imposed
425 perturbations. The final reported norms, in Table 1, for each technique are the mean
426 of the ten norms found for the application of the specific technique to each of the ten
427 datasets. The uncertainty is taken as the range of the norms over the ten datasets
428 and is also reported in Table 1.

429 Table 1 has two sections, with the norms of the upper section for application of the
430 QDC finding techniques (outlined in Sections 3.1–3.3) to the synthetic data, and the
431 norms of the lower section for the inclusion of the two step pre-smoothing method
432 (outlined in Section 3.4) prior to application of the techniques. The norms in each
433 section of the Table are arranged by technique and region of the Comparison matrix:
434 ‘All’ for the entire synthetic dataset, ‘Day’ for periods when the path is fully Sun-lit,
435 ‘Night’ for the periods when the path is fully dark, and ‘Twilight’ for the periods
436 when the day-night terminator intersects the path.

437 Lower norms result from technique calculated QDCs that are closer to the syn-
438 thetic background, on average. Thus the best technique is the one resulting in the
439 lowest norms. The norms in the top row of Table 1 compare the complete synthetic
440 dataset, including all the imposed perturbations, to it’s background. These norms
441 are the absolute upper boundary of what we would accept for the results from a QDC

442 technique as a higher norm would imply extra perturbations have been added by a
 443 technique.

444 The following subsections give the quantitative evaluation of the ‘Basic’ and ‘Pre-
 445 Smoothing’ techniques by their norms, as displayed in Table 1. Qualitative evaluation
 446 is provided for each technique by Comparison plots, i.e., the difference between the
 447 calculated and true QDCs. These plots are given in Figures 4 and 5 and are each
 448 processed from the same representative dataset of the ten used in the testing.

4.2. Evaluation of Basic Techniques

449 Comparing the norms within the upper section of the Table, we see that in the
 450 Twilight sectors the CDC and PCA QDCs result in higher norms than those for the
 451 synthetic dataset itself. In the Day and Night sectors, all three QDC techniques
 452 result in lower norms than those of the dataset. Across all sectors the FFT QDC
 453 finding technique shows the best results, with norms of less than 1 dB, whereas the
 454 CDC and PCA techniques both result in norms greater than 1 dB.

455 Figure 4 shows the Comparison plots for the synthetic data and all three basic QDC
 456 techniques. Plot (a), Data, is effectively just showing the synthetic perturbations,
 457 as expected. Plot (b) and plot (c), for the Basic CDC and Basic PCA techniques,
 458 respectively, show significant remaining influence of the imposed multi-day perturba-
 459 tions. Plot (d), Basic FFT, shows less localized influence of the perturbations than is
 460 seen in the plots for the other techniques. However, the overall effect of the imposed
 461 perturbations for this technique is to bias the calculated QDC in the dominant di-

462 rection of the data disturbance, either positive (day-time for the synthetic datasets)
463 or negative (night-time for the synthetic datasets).

464 The norms and Comparison plots for the three basic QDC finding techniques indi-
465 cate that the basic FFT technique is promising, but has the significant issue of bias,
466 which will be important in practical application. However, further investigations
467 found that these results can be significantly improved upon and the next section
468 gives the analysis for the addition of the developed pre-smoothing method to the
469 techniques.

4.3. Evaluation of Pre-Smoothing Techniques

470 The complete algorithm for each technique evaluated in this subsection involves
471 applying the two step pre-smoothing method, described in Section 3.4, to the full
472 synthetic dataset and then applying the chosen QDC finding technique to the result-
473 ing data matrix.

474 The top row of the lower section of Table 1 gives the norms for the comparison
475 of the smoothed synthetic dataset to the background. Here we see an immediate
476 improvement over all of the Basic norms in the upper section of the Table, excepting
477 only the Twilight norm for the FFT technique.

478 Applying either of the CDC and PCA techniques to the smoothed synthetic data
479 gives no improvement to the norms over the smoothing alone. Applying the FFT
480 technique to the smoothed data improves the results in all sectors, almost halving
481 the norms from the smoothing alone. The day-time norm for the pre-smoothed
482 FFT technique is 0.23 dB, which is around twice the maximum level of the day-time

483 imposed noise (0.1 dB). The night-time norm is 0.14 dB, which is less than half of the
 484 maximum level of the imposed night-time noise (0.5 dB). In contrast, the norm for the
 485 twilight-time section has increased compared with that of the basic FFT technique.

486 Figure 5 shows the Comparison plots for the smoothing method and pre-smoothed
 487 QDC finding techniques. Note the color scale range of this figure has been decreased
 488 from Figure 4. Plot (a), Smoothed Data, shows significant removal of perturbations
 489 from the calculated QDC, with only localized influence of highly perturbed periods
 490 in the synthetic dataset. Plot (b), Pre-smoothed CDC, shows no improvement over
 491 the Smoothed plot during the times of highly perturbed periods. The yellow and
 492 blue regions between 12 and 24 UT in the CDC plot show that a simple average of
 493 previous days as a QDC is prone to influence from any day-to-day slope present in
 494 the data, i.e., during Sun-lit periods in the synthetic dataset (14–24 UT, Figure 1c).
 495 Plot (c), Pre-smoothed PCA, shows the difficulty of separating background-related
 496 variance from perturbation-related variance in the PCA process. In this plot, the
 497 vertical sections encompassing the periods of twilight modal minima (23–04 UT and
 498 11–16 UT) show a distinct lack of definition for the minima while other sections are
 499 clearly influenced by the perturbations remaining in the smoothed synthetic data,
 500 such that they appear in our calculated QDC. The PCA QDC finding technique may
 501 have more success at identifying the true QDC for a shorter period dataset, of maybe
 502 month duration, however, investigation of this possibility is beyond the scope of this
 503 study. Plot (d), Presmoothed FFT, still shows some bias in the calculated QDC to the
 504 dominant direction of the data disturbance, however, this bias has been significantly

505 reduced from that seen in the Basic FFT Comparison plot of Figure 4. While the
506 pre-smoothed FFT technique does not represent the modal minimum periods well,
507 in general this technique provides the best calculated QDCs.

508 We conclude from the norms presented in Table 1, and examination of the plots in
509 Figures 4 and 5, that the best of the methods considered in this study for identifying a
510 QDC of a long-lasting VLF dataset, is to smooth the dataset as described in Section
511 3.4 then apply the FFT technique as described in Section 3.3. Unfortunately the
512 restriction of the FFT technique to datasets of at least two years duration, to allow
513 the row removal step to be applied, means that this technique is not appropriate for
514 shorter datasets. Thus, for datasets of less than two years duration we recommend
515 the pre-smoothing process alone as the best method for identifying a QDC.

516 Figure 6 shows a single representative day of synthetic data and the results for
517 the pre-smoothing process and the FFT QDC finding techniques. 6a is the synthetic
518 data and the two QDC results, which follow the diurnal pattern in the data visually
519 successfully. 6b shows the imposed perturbations for the day and the difference
520 between the data and each QDC, which we call the Remainder. At the visual level,
521 the Remainders contain the imposed perturbations. 6c shows the Comparison, which
522 is the difference between the true and calculated QDCs or equivalently between the
523 perturbations and the Remainder, for the two QDC results. For this day, the FFT has
524 larger magnitude Comparison values than the pre-smoothing process does in general,
525 however both lines on the plot remain within 0.25 dB of zero for most of the day.

526 We note that while the described methods give good results for identifying the QDC
527 from perturbations occurring during relatively slowly changing sections of data, such
528 as is usually seen when the VLF path is either fully Sun-lit or fully dark, the sharp
529 amplitude changes seen around the twilight modal minima times are not so well
530 dealt with. At this point we struggle to produce an accurate QDC representing the
531 intensity of twilight-time amplitude variations. Therefore caution is advised in the
532 interpretation of QDC finding technique results around the times of twilight modal
533 minima.

5. Application to actual AARDDVARK Datasets

534 We now provide example results of the application of this overall technique to our
535 AARDDVARK VLF datasets. We take the smoothing spans that were used for the
536 synthetic datasets and use these spans for the smoothing of the AARDDVARK VLF
537 datasets.

5.1. Clarifying the FFT Spectrum

538 When we began applying our FFT QDC finding technique to real VLF observations,
539 we found that the background-dominated central pattern of the FFT spectrum was
540 less distinct for some datasets than for the synthetic dataset. This lack of clarity
541 of the central pattern was identified as being caused by two sources. Firstly, the
542 dynamic range of amplitudes for a VLF dataset is usually much less, varying from 42
543 to 55 dB for the datasets used in this study, than the approximately 100 dB used for
544 the synthetic dataset. That value was set to ensure clear diurnal variations rather

545 than as an actual model of real VLF data. Secondly, the twilight-time modal minima
546 patterns in the synthetic dataset were based on a relatively short transmitter–receiver
547 path (NDK to EDM in Figure 1a at 1.304 Mm) and so had a very simple structure,
548 which made the background-related spectral patterns clear in the overall spectrum.
549 Longer paths demonstrate more complex twilight modal interference patterns due to
550 there being more distance along the path for interference fringes to occur [*Clilverd*
551 *et al.*, 1999]. The background-related spectral patterns in the spectrum are less clear
552 as the path lengthens, such as for the three Scott Base recorded transmitters in this
553 study.

554 In order for the parameters of the restriction stencil to be correctly identified when
555 the amplitude dynamic range is small and the modal interference patterns in the
556 dataset complex, the central pattern of the real VLF spectrum needs to be clarified.
557 We do this by subtracting an average magnitude row (found from the perturbation-
558 dominated higher frequency region of the spectrum) from the magnitudes of each row
559 of the overall spectrum, which leaves an approximate indication of the background-
560 related pattern in the spectrum for identification of the stencil boundaries. The
561 stencil is then applied to the “unclarified” spectrum as normal. With this addition
562 to the FFT QDC finding technique, the response of the real VLF datasets to the
563 technique improves.

5.2. Application Results

564 Figure 7 shows the dataset, calculated QDC, and the difference between the two
565 (Remainder) for 5 years of amplitude observations for the NWC (19.8 kHz) transmis-

566 sion received by the AARDDVARK antenna near Scott Base, Antarctica. The overall
 567 background patterns of the dataset appear well reproduced in the QDC. However,
 568 as the true QDC for real VLF amplitude observations is unknown, this is impossible
 569 to quantify. Some of the modal minima regions of the Remainder plot still show
 570 consistent amplitude differences, in contrast to the day-time and night-time regions,
 571 where the differences appear dominated by true perturbations.

572 Details from the Remainders in Figures 7c and 6b suggest that our FFT QDC
 573 finding technique is successful at identifying VLF responses to solar flares. This is
 574 confirmed for real VLF observations by examples of the VLF Remainder response
 575 to solar flares shown in Figure 8. These plots also show the, flare-defining, GOES
 576 satellite observed solar X-ray (0.1–0.8 nm) flux for the same period. The NWC-
 577 SB path was partly-lit until approximately 21:30 UT when it became fully Sun-
 578 lit, but still shows a visible response to the M1.7 flare, which occurs during the
 579 period of partial illumination. The other four paths were fully Sun-lit during the
 580 times of the shown solar flares. Variations in the solar X-ray observations outside
 581 of the flares are also seen as variations in the NLK–SB and NPM–SB observations.
 582 These examples demonstrate our QDC-finding technique’s success at identifying the
 583 underlying variation for relatively short-duration space weather events.

584 Figure 9 shows an example of a VLF response to a SPE for the NLK transmission
 585 observed by the Scott Base receiver. An SPE is defined for space weather purposes
 586 by the proton flux at energies >10 MeV exceeding a threshold of $10 \text{ (cm}^2 \text{ s sr)}^{-1}$ at
 587 geosynchronous orbit. The QDC in 9a shows a consistent diurnal variation, which

588 the amplitude data largely follows before the SPE begins and after the SPE flux
589 has returned to relatively quiet levels, i.e., approximately 80–96 hours in the plot. 9b
590 shows only the Day-time and Night-time Remainder. We do not show the Remainder
591 for the twilight modal minima periods in accordance with the caution advised for the
592 interpretation of the QDC during these periods. The Day-time Remainder shows a
593 clear offset from zero for the first two periods when the VLF path is Sun-lit after
594 the SPE begins. The Night-time Remainder shows a general offset from zero for
595 the first three periods after the SPE begins, although with more variability than the
596 Day-time periods show. Note that the SPE is clearly still affecting the data in the
597 third Night-time period even though the SPE flux is below the SPE threshold for this
598 period. 9c shows the corrected >10 MeV Proton flux observations from GOES-13
599 for context. The VLF amplitude response to changes in waveguide parameters varies
600 depending on the result of the superposition of multiple propagating modes. This
601 will not generally lead to a linear relationship between the perturbing SPE flux and
602 the observed remainder, as this figure shows. The remainder here demonstrates our
603 QDC-finding technique’s success at identifying the underlying variation even during
604 space weather events lasting multiple days. This figure also shows that the D-region
605 exhibits sensitivity to solar protons for fluxes below the SPE threshold.

6. Summary and Conclusions

606 In this paper we described three algorithmic techniques for the calculation of Quiet
607 Day Curves for observations of VLF transmissions propagated subionospherically.

608 1. The Combined Daily Curve technique calculated an average of the previous
609 three days' data for its QDC.

610 2. The Principal Component Analysis technique transformed the data matrix to
611 the directions of maximal variance, selected those directions accounting for more than
612 the mean variance and transformed them back to data-space for its QDC.

613 3. The Fast Fourier Transform technique transformed the data matrix to its
614 discrete spectrum, restricted those spectral components likely to be perturbation-
615 dominated, and transformed the restricted spectrum back to data-space for its QDC.

616 In addition, a smoothing process was described for application to the data prior to
617 a QDC finding technique.

618 We evaluated the success of these techniques at identifying the true QDCs of per-
619 turbed synthetic datasets and identified the algorithm combining the pre-smoothing
620 process (described in Section 3.4) and the Fast Fourier Transform based QDC find-
621 ing technique (Section 3.3) as the most successful technique on average over an en-
622 tire dataset. This combined technique was found to identify the true QDC of our
623 synthetic datasets to within 0.23 ± 0.02 dB during Day-defined periods and within
624 0.14 ± 0.01 dB during Night-defined periods of the datasets. The fast modal variations
625 during the Twilight-defined periods were identified to within 0.77 ± 0.05 dB. The FFT
626 based technique is only valid for datasets of at least two years, for shorter datasets
627 the pre-smoothing process alone, which was found to give the second best results in
628 the evaluation, is recommended as a QDC finding technique.

629 The combined pre-smoothing and FFT based QDC finding technique was then
630 applied to real datasets of observed VLF transmissions, from the AARDDVARK re-
631 ceivers located near Scott Base and Edmonton. Example results for five transmitter-
632 receiver paths were provided to demonstrate the technique's ability to identify re-
633 sponses to perturbations across the entire dataset (Figure 7), to solar flares (Fig-
634 ure 8), and to a multiple day SPE in real-world VLF data (Figure 9). From these
635 examples we deduce that this FFT based QDC finding technique will allow for sta-
636 tistical analysis of VLF responses to space weather events occurring in datasets of
637 longer duration than 2 years.

638 **Acknowledgments.** The synthetic dataset used throughout this paper and the
639 file used to create it are available from the corresponding author in MATLAB .mat
640 and .m formats respectively.

641 AARRDVARK VLF data availability is described at its website:

642 http://www.physics.otago.ac.nz/space/AARDDVARK_homepage.htm

643 The GOES-13 proton (>10 MeV) corrected flux data used in Figure 9c was down-
644 loaded (02/12/2014) from online file: [http://satdat.ngdc.noaa.gov/sem/goes/data/
645 new_avg/2011/11/goes13/csv/g13_epead_cpflux_5m_20111101_20111130.csv](http://satdat.ngdc.noaa.gov/sem/goes/data/new_avg/2011/11/goes13/csv/g13_epead_cpflux_5m_20111101_20111130.csv)

646 The GOES-14 X-ray data used in Figure 8a was downloaded (28/06/2014) from online
647 file: [http://satdat.ngdc.noaa.gov/sem/goes/data/new_avg/2010/01/goes14/csv/
648 g14_xrs_1m_20100101_20100131.csv](http://satdat.ngdc.noaa.gov/sem/goes/data/new_avg/2010/01/goes14/csv/g14_xrs_1m_20100101_20100131.csv)

649 The GOES-15 X-ray data used in Figure 8b was downloaded (12/11/2014) from online
650 file: http://satdat.ngdc.noaa.gov/sem/goes/data/new_avg/2010/01/goes14/csv/

651 g15_xrs_1m_20111101_20111130.csv

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Figure 1. (a) The great circle paths of the AARDDVARK observations analyzed in this study. Green circles indicate the locations of the monitored VLF communications transmitters, with call signs indicated. Red diamonds indicate the locations of the two AARDDVARK receivers (SB - Scott Base, Antarctica and EDM - near Edmonton, Canada). (b) Observations of the NDK transmission received at the EDM antenna from October 2011 to May 2014. (c) A synthetic dataset used for analysis of the success of our QDC finding techniques. White areas of plot (b) show the place-holder values replacing unusable data. The color-scales for the two upper plots are shown to the right of each plot. (d) Data from a representative day of the synthetic dataset. The red line is the true QDC, or background, and the black line is the complete data, combining perturbations and background.

Figure 2. Magnitudes of the 2-dimensional FFT spectra for the synthetic dataset shown in Figure 1c. (a) Basic spectrum before the restrictions are applied from the FFT QDC finding technique. (b) Fully restricted spectrum. Both plots have been zoomed in to frame the central background-related spectral pattern. The color-scale is \log_{10} and shown to the right of each plot.

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Figure 3. (a) Nearest neighbour distances between rows of the synthetic perturbation matrix versus the distances between the corresponding rows of the full synthetic dataset, perturbations and background combined. Vertical dashed line indicates top 10 % of full data distances. (b) Daily means for the full synthetic dataset (green line and markers), smoothed dataset (red line) and background of the dataset (black line).

Figure 4. Comparison matrices, i.e., the difference between the calculated and true QDCs, for the full synthetic dataset and three QDC finding techniques. The technique used to calculate the corresponding QDC is given in the top left of each plot. All plots are on the same color-scale, which is shown to the right of the plots.

Figure 5. Comparison matrices for the smoothed synthetic dataset and subsequent application of the three QDC finding techniques. The technique used to calculate the corresponding QDC is given in the top left of each plot. All plots are on the same color-scale, which is shown to the right of the plots and is smaller than that of Figure 4.

Figure 6. (a) Synthetic data for one day (black line) and the calculated QDCs found by the smoothing process (blue line) and FFT technique (red line). (b) Perturbations in the dataset and the remainders from the techniques. (c) Comparisons between the calculated and true QDCs. All three plots have a guide bar as to the level of light on the path, either fully sunlit (light-grey), fully dark (dark-grey), or mixed with the terminator located across the path (mid-grey). The date of the day is given in the x-axis label to allow cross-checking with Figure 1c.

Figure 7. (a) Observations of the NWC transmission received at the SB antenna from January 2009 to December 2013. White areas of the plot show the place-holder values replacing unusable data. (b) The QDC calculated using the pre-smoothed FFT technique. The color-scale for the dataset and QDC plots is given to the right of the QDC plot. (c) The remainder, or difference between the dataset and the calculated QDC, with color-scale to the right of the plot.

Figure 8. (a) Remainders (observed amplitudes - calculated QDC) for three transmitter signals observed by the Scott Base receiver (solid colored lines, left y-axis) for 17–24 UT on 19 January 2010. (b) Remainders for two transmitter signals observed by the Edmonton receiver for 17–24 UT on 5 November 2011. Included on the plot are solar X-ray observations (thick dashed black line, right y-axis) from (a) the GOES-14 satellite and (b) the GOES-15 satellite. Grey dashed horizontal line indicates 0 dB remainder, i.e., where the calculated QDC equals the data. Grey dashed vertical lines indicate the peak flux times for NOAA identified solar flares, with the magnitude of each flare given at the base of each line.

Table 1. Norms (Equation 4.1) for the comparison of our calculated QDCs to the true QDCs of our synthetic datasets. All values are rounded to 2 decimal points. Units are dB.

	All	Day	Night	Twilight
Data	2.65 ± .08	1.71 ± .04	3.45 ± .14	1.74 ± .15
CDC	1.48 ± .03	1.24 ± .01	1.33 ± .08	2.86 ± .01
PCA	2.53 ± .08	1.59 ± .04	3.24 ± .14	2.36 ± .10
FFT	0.85 ± .01	0.82 ± .01	0.9 ± .02	0.68 ± .01
	All	Day	Night	Twilight
Smoothed	0.57 ± .03	0.44 ± .05	0.24 ± .02	1.61 ± .14
CDC	0.96 ± .02	0.64 ± .02	0.29 ± .01	2.95 ± .05
PCA	0.79 ± .04	0.54 ± .03	0.47 ± .02	2.17 ± .12
FFT	0.28 ± .01	0.23 ± .02	0.14 ± .01	0.77 ± .05

Figure 9. (a) Data from the NLK transmission observed by the Scott Base receiver (black line), and the calculated QDC (red line) for the period of a SPE starting 26 November 2011. The background color indicates the level of light on the path, either fully sunlit (light-grey), or with the terminator located across the path (mid-grey). (b) Remainder during periods when the path is fully Sun-lit or mostly dark, with the background color indicating the light level. (c) Corrected >10 MeV Proton flux observations from GOES-13. The y -axis of this plot is a \log_{10} scale. The threshold for SPE recognition is marked by a horizontal dashed black line. In all plots the dashed vertical blue line indicates the time of onset of the initial flux increase, the green line the time when the SPE threshold was exceeded, and the red line the time of peak proton flux.

















