Low-cost contour and groundwater mapping

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Abstract: Faced with the need to establish contours for a sloping quarter acre section, I developed a simple and low-cost way of making a survey from which accurate contours can be produced. The solution requires measuring the distance between suitably-placed survey pegs, and also their relative heights, followed by computation of a sizable inverse problem. A groundwater problem on a neighbouring property prompted me to also develop a low-cost piezometer for measuring the water level in bore holes. The surveying method is described with the resulting plan of contours, and the piezometer circuit and details are given.

Keywords: Contour mapping, surveying, inverse problem, groundwater, fluid sensor, piezometer

1. INTRODUCTION

In 2008 we[1] bought a quarter-acre section in North Dunedin. Being warm-blooded immigrants from the north we chose a section that catches lots of sun, which in Dunedin means that the section is on a hill side. (In Dunedin land on the flat is either in the valley, which gets the frost, or on the ridge, where it can be very windy.) A picture of the land, and slope is in Figure 1.

Figure 1: A sunny north-facing slope in Dunedin. The slope can be seen to be roughly 1 in 3.

With an intention to build, I decided to survey the land to provide contours to our architect. I first went the high-tech route using GPS that allows post processing to sub-metre accuracy. That allowed absolute placement of the section pegs, but I found the process too slow and imprecise for the contouring job. In the end I settled on a low-tech approach using a spirit level made from a garden hose, a tape measure, and string lines to fill in detail as needed. Measurements are then quite quick and accurate, but leave a quite large implicit problem to determine the position of pegs and hence contours. The measurement equipment, procedure, data protocols, and structure of the inverse problem are covered in section 2.

Our house design would require excavation to a depth of 3 metres, so we became concerned about the possibility of groundwater flow to the house site, both during construction and after the house was built. We are aiming at building a well-insulated house; contrary to popular belief, Dunedin actually gets plenty of regular sun. So it is feasible to build a house that maintains an internal temperature around 20 degrees C, without any heating at all [1][2]. The key is thermal mass and insulation – the combination stores the heat from the sun for the days when there is none.

We had experienced this system when living in Guanajuato in México, which (contrary to popular belief) is often a cold place. There, the houses are built against the bedrock which stores the heat from the day, keeping the house warm during the very cold nights. Only after three days of overcast weather did our house actually need heating.

[1] Andrea and I

[2] We will put in a log burner for those times when theory and practice don't match.

Guanajuato has the advantage of being incredibly dry, which aids thermal insulation.

Water flow against a building is not conducive to maintaining warmth (or dryness). To measure the groundwater on the site, I enlisted the help of a soil geologist friend. We bored three 3-5 metre deep bore holes and measured the level of water. The first measurements were made using an industrial water-level sensor that we borrowed. For further measurements I built a fluid-level sensor that give the same functionality, using bits out of my electronic junk box. The electronics and sensor are described in section 3.

2. LOW-COST SURVEYING

2.1 Low cost GPS surveying

My first intention was to survey the land solely using some cheap global positioning system (GPS) gear. Standard GPS equipment has an accuracy of about 10m, which is obviously not good enough for determining contours. Professional surveying GPS gear uses a differential system that requires a local base station to transmit it's own location, so that a moving station can subtract out most of the errors in the system. This still leaves something like 1 metre accuracy, but can be fast. However, these systems cost something like \$100k, so are well outside my budget.

Instead I opted for some affordable gear from DeLorme [2] that allows post-processing of the data to give sub-metre accuracy. The units I bought were an Earthmate Blue Logger GPS, and an Earthmate USB GPS, costing about NZ\$300 all up. That allowed me to log one GPS directly to my laptop, while the other logged itself, for later download via Bluetooth. The gear worked well, though with a few teething and software problems and the usual Bluetooth issues, and did indeed allow sub-metre measurements. I found that very useful for accurately locating boundary pegs with respect to the survey reference points in Dunedin. However, the post-processing essentially works by *averaging* a long record to reduce errors. In practice that means recording for an hour or two, at a time when many satellites are overhead. DeLorme provided great software for finding out in advance when a suitable satellite configuration was overhead, but that did mean planning measurements in advance to fit them around other commitments. In practice I could manage a few measurements per day which was simply too slow for the surveying job. On top of that, accuracy around 1 metre was not really good enough for the half-metre contours that the architect wanted. So I turned to the low-tech route, described next.

2.2 Low-tech surveying, in theory

The GPS route had created the hope that I could determine the absolute location of any point. With that easy route gone, I decided to measure the relative location of selected points by the tried-and-true method of triangulation. I fixed a set of points by putting in pegs at regular locations, with a clout-head nail in the top of each peg to give a precise measurement point.

To locate the pegs in three dimensions requires measuring the relative heights, as well as relative distances. Consider three points in space $r_1 = (x_1, y_1, z_1)$, $r_2 = (x_2, y_2, z_2)$, and $r_3 = (x_3, y_3, z_3)$. The relative heights $h_{1,2} = z_1 - z_2$, $h_{2,3} = z_2 - z_3$, and $h_{3,1} = z_3 - z_1$ and distances $d_{1,2} = \|r_1 - r_2\| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$, $d_{2,3}$, $d_{3,1}$ are all that is needed. For these three points, the relative z-values are determined by $h_{1,2}$, $h_{2,3}$, and $h_{3,1}$ (note there is a redundancy since $h_{1,2} + h_{2,3} + h_{3,1} = 0$). The x and y values can be found by writing $(x_1-x_2)^2 + (y_1-y_2)^2 = d_{1,2}^2 - h_{1,2}^2$, etc, and solving for three of the six unknowns. The position of the three points is then determined up to an arbitrary translation in 3-dimensions, a rotation in the horizontal plane, and a mirror symmetry of the horizontal plane[3].

The location of further pegs can be found by further triangulation. For example, if we include a peg at r_4 and measure distances to two previous pegs and one relative height, the three measurements determine the three coordinates (x_4, y_4, z_4), up to the mirror symmetry.

The translation and rotation ambiguities may be resolved by choosing one peg to be at the coordinate origin (determining three values) and by determining the orientation of one line with respect to north. I did this at the final step, by determining the absolute location of two of the boundary pegs. The mirror ambiguity is resolved by knowing the approximate location of pegs, and picking the closest solution.

2.3 Measuring distance and height

Measuring distance is easy with a tape measure, but measuring relative heights requires a bit more ingenuity.

My first thought was to use a hose filled with water and measure the pressure drop across the hose, and hence determine relative heights of the two ends. I soon found that pressure transducers with

[3] There are *three* mirror symmetries, but the result depends only on whether an odd or even number of mirror operations are applied.

the required accuracy are a bit pricey, so I opted for a simpler approach. I made a simple spirit-level using a garden hose with about a metre of clear plastic hose attached at each end. A ball tap at each end allowed the ends to be closed so the water did not leak out while moving the hose, and could be opened to the atmosphere when making measurements – so the water at both ends have the same (atmospheric) pressure and hence are at the same height. I measured relative heights by mounting a pole of known height vertically over each peg, and measuring the height from the water level to the top of the pole. I squirted some dish-washing liquid into the water in the pipe to reduce any affects of surface tension. That had the unexpected benefit of creating foam on top of the water, making it easy to see the water level. The hose level is shown in Figure 2, with both ends clamped to a pole placed over a peg. The pole has a short section of pipe attached to the bottom, to allow easy placement over the peg, and a post-level attached to make it easy to ensure the pole is vertical.

Figure 2: The hose level made up of a water-filled garden hose with clear tube ends, with both ends clamped to a post. A red post-level is visible, while foam on top of the water is just visible.

I also used a string line between pegs to fill in surface levels, as needed. I used thick fishing line as the string line; being light and strong it can be pulled tight and does not droop much. Measurement of the distance from one peg determines the 3-dimensional location of a point on the string line, and the height above the surface was recorded.

2.4 Measurement protocol, parsing, and data structures

Measurements were initially written (in pencil) in a notebook, as they were made. In total, several hundred measurement were made and by the end I wished I had an electronic way of recording measurements without the need to write. I came up with a simple readable space delimited text file format for entering this data into the computer, that kept the process simple and easy to check.

All data was entered into one text file, with one measurement per line. The lines have the format

- `H peg1 peg2 hhh` – height `hhh` cm of peg1 above peg2
- `D peg1 peg2 ddd` – distance `ddd` cm between peg1 and peg2
- `SL peg1 peg2 ddd hhh` — the height `hhh` (cm) above ground of a string line a the point distance `ddd` cm from peg1

The terms `peg1` and `peg2` were the names given to the pegs. At the time of putting them in, each peg was marked with a short name such as ‘L1’ for pegs establishing basic levels, ‘BP4’ for pegs on the boundary, and ‘C3’ for fill-in pegs needed for more accurate contours.

The file was first parsed to build a table of peg names, and a mapping from names to numbers. Two arrays were constructed for each of the height and distance data. Array H was a measurement mask for heights with $H(i, j) = 1$ if the height between the i^{th} and j^{th} pegs was measured, and is otherwise zero, with the height array h storing the value of $h_{i,j}$. Note that H has even symmetry while h has odd symmetry. Similarly array D was a measurement mask for distances with $D(1, j) = 1$ if the distance between the i^{th} and j^{th} pegs was measured, and is otherwise zero, with the distance data array d storing the value of $d_{i,j}$. Note that D and d have even symmetry.

2.5 An inverse problem

Each of the raw distance or height measurements has an accuracy of about ±0.5cm, from reading the tape measure. Once geometry inaccuracies are accounted for, such as sag in a tape measure line or wobbly pegs, the errors are about 2 or 3 times that, and maybe more in long distance measures where tape measure stretch is an issue.

These errors mean that the positions of pegs cannot be determined by exact solution of the equations, above. Attempting that route would probably end up with massive errors by the time the last measurements were being analyzed. Further, redundancy in the measurements (which ought to improve results) in the presence of errors means there is no set of peg positions that exactly satisfy the distance and height equations. So what to do?

This is a common situation in inverse problems. A standard solution is to solve *all* of the equations at once, but only approximately. And how to do that? The late great Murray D. Johns once said that solving any problem is easy ... just pick a criterion of optimality and optimize with respect to it[4]. So that is what we do here, by minimizing a sum of squares of the data residuals, as follows.

Using the arrays defined in section 2.4, define the square error in heights

$$Q_h = \sum_{H_{i,j}=1} (z_i - z_j - h_{i,j})^2$$

and the square error in distances

$$Q_d = \sum_{H_{i,j}=1} (\|r_i - r_j\| - d_{i,j})^2$$

which are functions of the peg positions. The equation in heights decouples so it is easy to solve for heights first. Since Q_h is quadratic in the heights, the set of z-values that minimize Q_h is given by solving a linear system involving the Hessian of Q_h which is (in MatLab notation) `-H+diag(sum(H))` – it is no coincidence that this has the same form as the admittance matrix for a network of 1 Ohm resistors. Fixing these z values, I then solved for the x- and y-positions by minimizing Q_d. This functional is not quadratic in peg positions so requires a bit more work to minimize. I took the lazy route of using MatLab's `fmins` function, which takes ages to converge but does get there. It is important to resolve the translational and rotational symmetries to avoid unnecessary numerical work, and to choose a starting position that resolves the mirror symmetries.

Once peg positions were determined, I used a radial basis function interpolation to evaluate the ground level at any point. Figure 3 shows the contours determined this way, with the location of pegs shown as blue crosses, plotted on an aerial photograph downloaded from the Dunedin City web site.

[4]It turns out this suggestion of making a 'point' estimate can sometimes give terrible predictions. See the other paper co-authored by me in this proceedings for some examples that demonstrate this.

3. MEASURING GROUNDWATER

Underground water was probed by drilling bore holes, and measuring the water level in each bore hole over time.

With the help of a friend who is a soil geologist, we borrowed a 60mm diameter hand auger that had extensions to allow bore holes of several metres depth. In the end we bored three holes, two at the location that will be the back wall of the house (and the deepest excavation) and one near an area where a neighbour had experienced problems from excessive groundwater. The bore hole varied from 3.5 to 4.5 metres – basically we kept going until we hit a rock and could not bore further.

I put 30 mm diameter stand pipes in each borehole, with horizontal hacksaw cuts each 50mm for 1.5 m from the bottom to allow water flow, then covered in filter material to stop silting. The bore hole around the stand pipe was back-filled with coarse sand to stop the hole collapsing, which can also block flow. Figure 4 shows one stand pipe.

Figure 4: All that can be seen from the surface of one stand pipe

3.1 A junk-box piezometer

An easy way to measure water height is to use a 'piezometer'. A sensor is lowered into the standpipe, and the piezometer indicates when the sensor touches water. The sensor head is essentially a pair of electrical contacts with the water closing an electrical circuit.

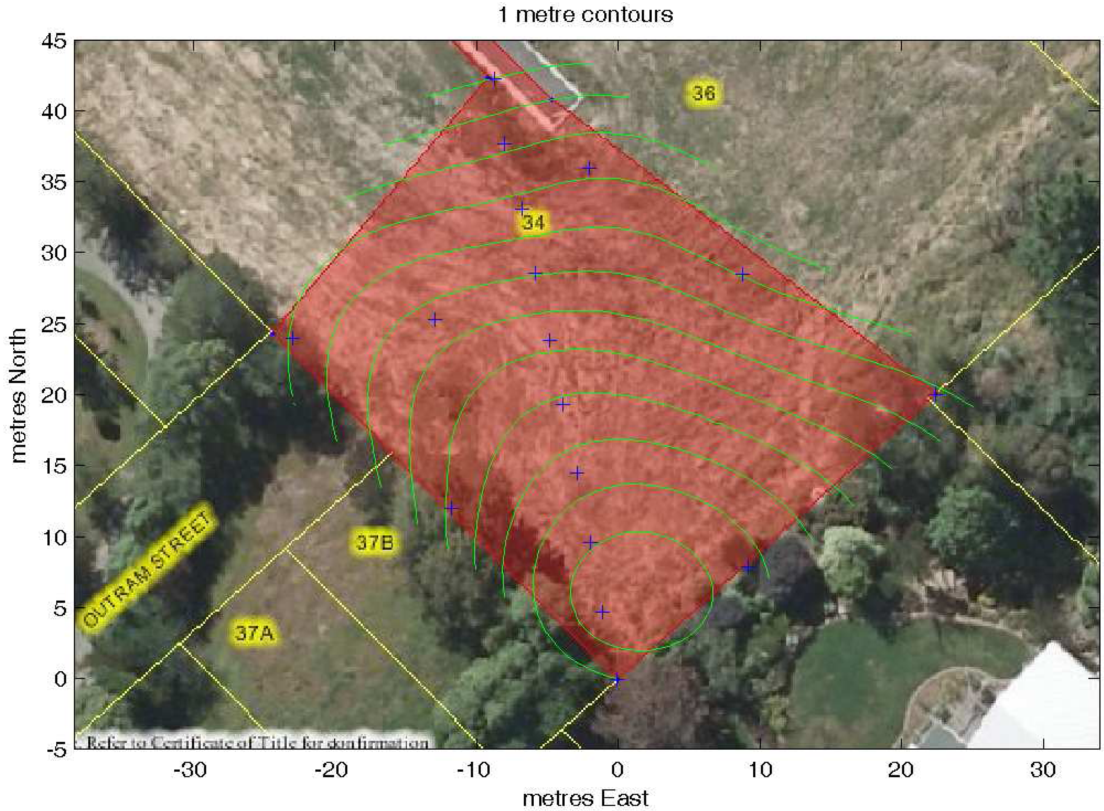

Figure 3: Contours, in green, interpolated from 3-dimensional peg positions, shown as blue crosses.

On the day we bored holes, we used a professional piezometer that we borrowed to measure the height of the groundwater. For subsequent measurements I built a unit that mimicked the operation of the professional unit. Figure 5 shows

Figure 5: The sensor head connected to a tape measure and wiring, sitting on the box that houses the electronics.

the sensor head I built, made from a stainless steel center rod with a 10mm long 3mm diameter probe turned at the bottom end, wrapped in insulating tape and shoved into a outer tube, with 10mm cross hole at the probe. The outer tube turned out not to be stainless, as can be seen from the surface rust.

Measurement of electrical connectivity at the sensor is best made at frequency, as opposed to d.c., to avoid electrolysis effects that can build up an insulating layer on the sensor. I chose a frequency of 1.2 kHz.

The electrical equivalent of the sensor at 1kHz is shown in Figure 6, when dry and when dipped in tap water. Not surprisingly, when dry the sensor

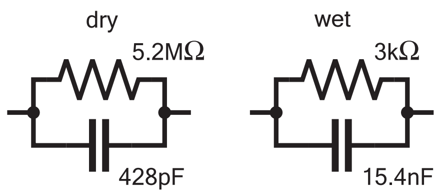

Figure 6: The electrical equivalent of the sensor head, wet and dry, measured on a RCL bridge at 1kHz.

is essentially an open circuit with the leads contributing about 430pF, so the impedance is primarily capacitive. When wet, the impedance at 1kHz is primarily resistive, looking like a 3kΩ resistor with some capacitance.

The water sensing circuit is shown in Figure 7, adapted from a circuit I found on the web [3]. This gives a device that gives an audible beep when

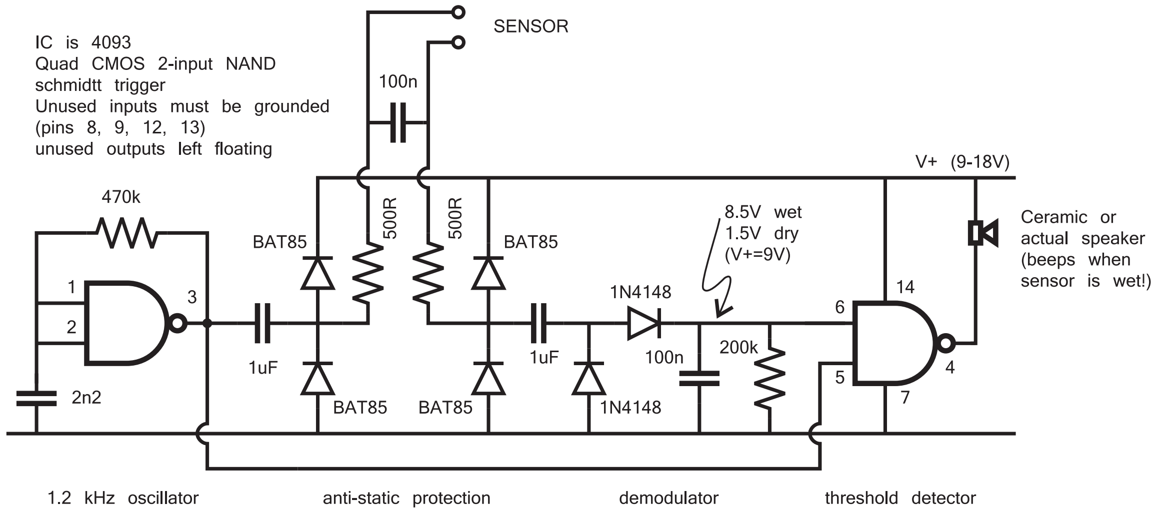

Figure 7: Circuit diagram for the piezometer.

the sensor is wet. It uses half of a CMOS 4093 quad two-input NAND schmitt trigger. I have to say that I'm a fan of this very versatile chip, and recommend the Fairchild application note [4]. I added the anti-static protection [5] after zapping the first version within a day. All the parts required were retrieved from my junk box. The speaker is a piezo transducer (probably scavenged from a musical Christmas card that had become annoying) glued to the inside of the box. The whole thing runs from a 9V battery and requires no on/off switch as it draws virtually no power when not beeping. With only the connecting terminals going through the box, I could seal the box completely making the whole unit weather proof. The unit I built is six months old now, and still beeps happily when the sensor is wet.

4. DISCUSSION AND CONCLUSION

I have now surveyed two plots of land using the method described here. One house is built using the resulting contours; unfortunately it's not ours. However, as we have shown, it is perfectly feasible to make contours of a quarter-acre plot with some $100 worth of bits from the local hardware store, followed by several hours of high-tech computing courtesy of MatLab.

I was also surprised how easy it is to measure groundwater, and to build a piezometer. A hydrologist friend is now interested in replicating and using the design I have given here to save the thousands it costs to buy the professional unit.

As part of the groundwater measurements I also built a simple water pump for performing 'pump tests'. Electrical pumps that can suck muddy water up 4 metres are rather expensive, and rely on a purpose-made non-return valve at the bottom of the bore. In keeping with the cheapo aspect of this project, I built a simple pump following a design I seen used in poor regions of Africa. It consists of a ball valve at the bottom of a long pipe; oscillating the pipe up and down pumps the water up. I made one using some 25 mm water pipe from the local hardware store, a few adapted fittings, and a marble. I was amazed how well it worked, and how good the valve turned out to be. This allowed me to perform a 'draw down' test to determine water availability – again requiring much processing in MatLab.

My conclusion is that a lot can be done with a few parts from the hardware store, an electronic junk box, and lots of MatLab processing.

5. REFERENCES

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