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In Situ Measurement of Power Flow and Mechanical Properties of Vibrating Timber Structures.

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Abstract

Determining vibrational power flow in a structure requires knowledge of local mechanical properties as well as measurement of the vibrational motion. By using more measurements than are required to measure the motion, we model-fit to find the local mechanical properties and hence determine directional power flow. We comment on the selection of optimal transducer placement for greatest measurement accuracy, and give an example of measuring directional bending-waves in a wood beam where the dynamic mass and rigidity are unknown *a priori*.

1. Introduction

The New Zealand building industry uses timber framed constructions extensively in smaller sized buildings, and this has many advantages in speed and economy of construction and in use of an indigenous renewable resource. However, sound transmission can be a problem because of the light weight and stiffness of structures and this detracts from its use particularly in multi-residential buildings.

This work is part of a project to measure and model sound transmission in timber framed structures. We treat timber elements, such as studs and joints, as simple beams, to develop a method for measuring the applicable dynamic properties. A theoretical model is used for the bending modes of the beam, which are most important in acoustic radiation. The relevant dynamic properties of pinus radiata (the most common structural timber) have been measured and found to be close to but not identical to the static properties.

2. Bending Waves in Beams

Under the usual thin-beam assumption the transverse displacement, $\eta(x,t)$, is related to the applied pressure, $p(x,t)$, via [1]

$$B\eta_{xxxx} + m\eta_{tt} = p, \tag{1}$$

where x is the coordinate along the beam, t denotes time, and a subscript denotes differentiation with respect to the variable. Here B is the bending stiffness of the beam, and m is the mass per unit length. Note that both B and m are usually taken to be constants, though for practical materials the *effective* values can vary spatially and with the rate of bending.

The modes of the beam have the form $\mathrm{e}^{i(kx+\omega t)}$. In sections of the beam where no external pressure is applied we have $p = 0$ and the only allowable solutions have k and ω related by $k = (\omega^2 m/B)^{1/4}$. At fixed radial frequency, ω, the displacement takes the form (the real part of)

$$\eta(x,t) = \mathrm{e}^{i\omega t}\left(a_1\mathrm{e}^{ik_\omega x} + a_2\mathrm{e}^{k_\omega x} + a_3\mathrm{e}^{-ik_\omega x} + a_4\mathrm{e}^{-k_\omega x}\right), \tag{2}$$

where k_ω is the positive real allowable root. We will denote the expression in the parentheses by $\eta(x,\omega)$, which is the complex coefficient of $\mathrm{e}^{i\omega t}$. The modes with coefficients a_1 and a_3 are travelling waves, propagating towards $-\infty$ and $+\infty$, respectively, while the other two modes are evanescent and decay away from the location of forcing or joints in the structure.

2.1 Power flow

The power transported by the wave travelling towards $-\infty$ is [1] $P = |a_1|^2 B\omega k^3 = |a_1|^2 B^{1/4} m^{3/4}\omega^{5/2}$. Note that calculation of power requires the amplitude a_1, the bending stiffness B, the mass density m, as well the (known) frequency.

3. In Situ Measurement

Suppose the transverse displacement is measured at N positions, $x_1, x_2, \cdots, x_N$, giving the N time series $\eta(x_l,t)$ for $l = 1, 2, \cdots, N$. The Fourier transform of each time series gives the complex amplitudes $\eta(x_l,\omega)$ for $l = 1, 2, \cdots, N$, at each position and radial frequency, ω. We actually measure the transverse acceleration, hence the Fourier transform gives $-\omega^2\eta(x_l,\omega)$, though this does not fundamentally affect our calculation. Given the set of measured complex amplitudes, the various unknown parameters and amplitudes may be found by fitting the functional form of $\eta(x_l,\omega)$ to the measured values.

3.1 Estimating modal amplitudes

Consider the simple case where the ratio m/B is known (perhaps by static measurement) and we wish to find the 4 modal amplitudes a_1, a_2, a_3, a_4. This may be done using standard model fitting methods [2], as follows.

Write E as the $N \times 4$ matrix

$$E = \begin{pmatrix} e^{ik_\omega x_1} & e^{k_\omega x_1} & e^{-ik_\omega x_1} & e^{-k_\omega x_1} \\ e^{ik_\omega x_2} & e^{k_\omega x_2} & e^{-ik_\omega x_2} & e^{-k_\omega x_2} \\ \vdots & \vdots & \vdots & \vdots \\ e^{ik_\omega x_N} & e^{k_\omega x_N} & e^{-ik_\omega x_N} & e^{-k_\omega x_N} \end{pmatrix}, \tag{3}$$

the vector of coefficients as $a = (a_1, a_2, a_3, a_4)^T$, and the vector of measurements as $y = (\eta(x_1, \omega), \eta(x_1, \omega), \cdots, \eta(x_N, \omega),)^T$. The relationship between amplitudes a and (noise-free) measurements y is then $Ea = y$, so finding the coefficients a requires solution of this matrix equation.

The matrix E is generally invertible when 4 measurement positions are used, i.e., $N = 4$. While that case provides a simple theoretical route, it is seldom practical as E can be ill-conditioned over a range of frequencies, causing measurement imprecision to lead to spurious estimates of the amplitudes.

The accuracy of estimates is always improved by using more than 4 measurement locations. Since E has rank at most 4, and y then has $N > 4$ components, measurement and modelling errors will cause y to lie outside the range of E. Modelling errors arise because the timber beams are never precisely homogeneous and measurement positions contain error. We only explicitly consider measurement error and assume that the modelling error can be treated within the same framework. We assume that measurement error is additive, i.i.d. with zero-mean normal (Gaussian) distribution and variance σ^2. That is, $Ea = y + n$, where $n = (n_1, a_2, \cdots, n_N)^T$ with each $n_l \sim N(0, \sigma^2)$. Then a can be found by maximum likelihood estimation (equivalently, $Ea = y$ solved in the least-squares sense). Since the matrix E will still have small (though non-zero) singular values, it is prudent to regularize [3] giving the estimate

$$\hat{a} = \left(E^H E + \alpha I\right)^{-1} E^H y, \tag{4}$$

in which ^{H} denotes conjugate transpose, I is the 4×4 identity, and α is a small positive regularizing parameter. There is an extensive theory on how to set the value α [3], however we simply use $\alpha = \|E^H E\|_2 N\sigma / \|y\|_2$ which suppresses the effects of noise while otherwise allowing accurate estimates of the coefficients.

3.2 Estimating the mechanical properties

When measurements are made at 5 or more locations, we may extend the method given above to include estimation of the ratio m/B that appears in the expression for wavenumber. We do this by adjusting the value of m/B, as well as the four coefficients a_1, a_2, a_3, a_4, to minimize the square misfit $\|E\hat{a} - y\|_2^2$ between the predicted measurements, $E\hat{a}$, and the actual measurements, y. Since, for a given value of m/B the best fit is achieved using equation 4, we may find the best value of m/B by adjusting k_ω appearing in the definition of E to minimize $\|E\left(E^H E + \alpha I\right)^{-1} E^H y - y\|_2^2$.

While the ratio m/B is determined by the fitting procedure, a second measure is required to obtain the values B and m separately. Since we use point forcing of the beam for the ratio of force to amplitude of the outward travelling wave depends on the factor $m^3 B$, measuring this ratio allows both parameters to be determined.

3.3 Optimal measurement location

The misfit $\|Ea - y\|_2^2$ is proportional to the log-likelihood [4] of the parameters given measurements y. A measure of the amount of information in the measurements about the parameter m/B is given by the Fisher information measure [4], which, since the noise is assumed Gaussian, is $\|\partial/\partial(m/B)Ea\|_2^2/\sigma^2$. Optimum measurement geometry is found by maximizing this number over feasible measurement positions.

4. Experimental Results

We measured the bending waves in a 2.8 m length of 100 mm $\times$ 50 mm dry pinus radiata, mounted between near-anechoic sand traps. This beam was centrally driven by a point source with power between 100 Hz and 3 kHz. The resulting transverse acceleration was measured at distances (close to) 400 mm, 600mm, 800 mm, 900 mm, and 1000 mm from the forcing.

By fitting the modal amplitudes and m/B as above, we determined: The amplitude of the wave reflected from the sand trap was about 10 % of that entering the trap, hence about 99 % energy absorption is being achieved. The value of B/m decreases from 2.5×10^3 Nm^2 at 100 Hz to 1.45×10^3 Nm^2 at 3 kHz, roughly linearly with frequency. This compares with the value 2.65×10^3 Mm^2 measured statically.

Conclusions

We have shown that both modal amplitudes and mechanical properties can be estimated from measurements of transverse motion. Using this method we found that the mechanical properties of pinus radiata vary with frequency and, hence, accurate measurement of power flow cannot rely on the statically measured value of m/B.

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